1. Sketch the region enclosed by the curves $y = x^2 - 2x$ and $y = 3x$. Find the area of the region. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle.

   OK thank you Desmos

   Or if you prefer thank you Wolf

   Notice that wolfram gives you the answer you were looking for, but I could always ask you to do the steps, so lets do them quickly.

   It is clear that the curves meet at $x = 0, x = 5$ by solving the quadratic equation $x^2 - 2x = 3x$. That gives the limits of integration. The upper curve is the line $y = 3x$ the lower is the parabola $y = x^2 - 2x$ subtract the lower from the upper and get

$$\int_0^5 5x - x^2 dx$$

   a very easy integral. The anti derivative is

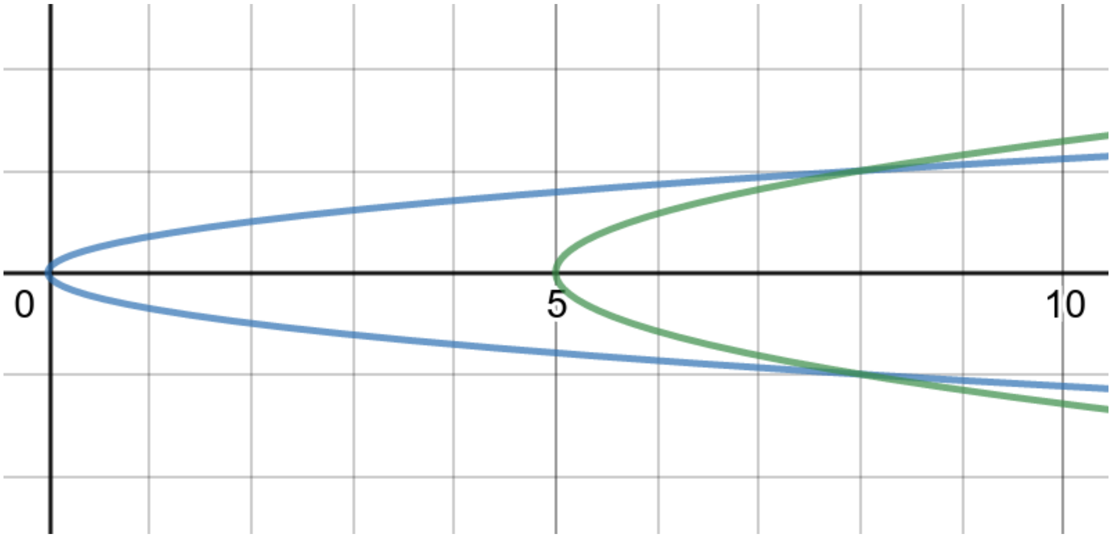
$$F(x) = \frac{5x^2}{2} - \frac{x^3}{3}$$

   compute

$$F(5) - F(0)$$

   although really just $F(5)$ since $F(0) = 0$

2. Sketch the region enclosed by $x = 8y^2, x = 5 + 3y^2$ and find the area.



Clear we want to integrate with respect to y since x is written as a function of y and we want to go from bottom to top not left to right. To find the limits of integration set

$$8y^2 = 5 + 3y^2$$

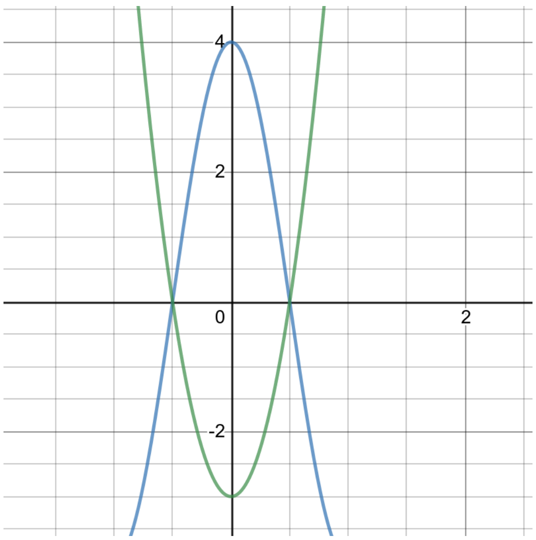
solve and get $y = -1, y = 1$ The curve on the right is $x = 5 + 3y^2$ on the left $x = 8y^2$ therefore the area is

$$\int_{-1}^{1} \left(5 + 3y^2 - 8y^2\right) dy = \int_{-1}^{1} \left(5 - 5y^2\right) dy = \int_{-1}^{1} 5dy - 5 \int_{-1}^{1} y^2 dy$$

$\int_{-1}^{1} 5dy = 5 \times 2 = 10$

$$-5 \int_{-1}^{1} y^2 dy = -5 \left. \frac{y^3}{3} \right|_{-1}^{1} = -5 \left(\frac{1}{3} - -\frac{1}{3} \right) = -5 \times \frac{2}{3} = -\frac{10}{3}$$

3. Sketch the region enclosed by $y = 4\cos(\pi x), y = 12x^2 - 3$



Both cross the x axis at $\frac{1}{2}$ by inspection. The top curve is $y = 4\cos(\pi x)$ the lower one is the parabola $y = 12x^2 - 3$ So we can find the areas between then by

$$\int_{-.5}^{.5} \left(4\cos(\pi x) - 12x^2 + 3\right) dx$$

Not hard to compute but Click Here. Or, if you really want to do nothing, use This

4. Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line. $y = x + 1, y = 0, x = 0, x = 2$ about the x-axis

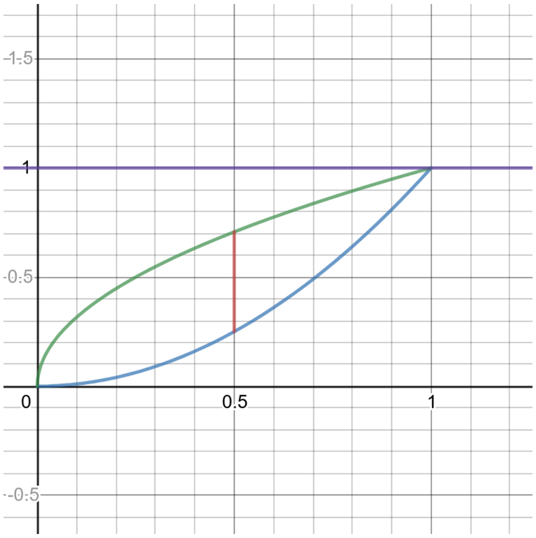
I used to hate this, but it is not hard, and we will make it even easier. The idea is that the cross section of the volume will be a circle with area πr^2 only r isn't fixed. In this case r is the function $r = x + 1$ so the volume will be

$$\int_0^2 \pi(x+1)^2 dx = \pi \int_0^2 \left(x^2 + 2x + 1\right) dx$$

and easy enough integral to compute. The picture (and the answer) is Here

5. Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line. $y = x^2, x = y^2$ about $y = 1$

The picture of the area looks like



It is going to be revolved around the line $y = 1$ so the outer radius will be the distance between 1 and x^2 which is $r_1 = 1 - x^2$ The inner radius will be $r_2 = 1 - \sqrt{x}$ The volume will be

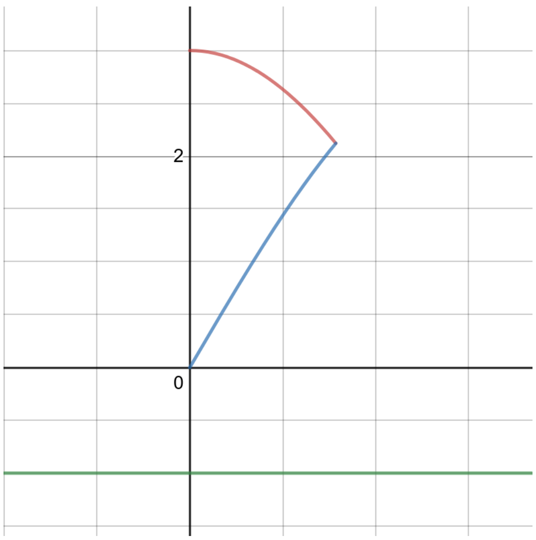
$$\pi \int_0^1 r_1^2 - r_2^2 dx = \pi \int_0^1 \left(1 - x^2\right)^2 - \left(1 - \sqrt{x}\right)^2 dx = \pi \int_0^1 \left(-2x^2 + x^4 + 2\sqrt{x} - x\right) dx$$

The last equal sign after a bunch of algebra, all doable, all annoying. This problem is slated to take 26 minutes according to WebAssign. It should take two

6. Find the volume V of the solid obtained by rotating the region bounded by the given curves about the specified line.

   $y = 3\sin(x), y = 3\cos(x), 0 \le x \le \frac{\pi}{4}$

   Looks like

   The top curve is $y = 3\cos(x)$ the bottom is $y = 3\sin(x)$ and they are being revolved about $y = -1$ so the outer radius will be $r_1 = 1 + 3\cos(x)$ the inner $1 + 3\sin(x)$ and the integral

$$\int_0^{\frac{\pi}{4}} (1 + 3\cos(x))^2 - (1 + 3\sin(x))^2 \, dx = \pi \int_0^{\frac{\pi}{4}} (-6\sin(x) + 6\cos(x) + 9\cos(2x)) \, dx$$

   I could get the picture from wolfram

Two more volume questions.

7. Find the Volume of a pyramid with base an equilateral triangle of side a and height h. First note that your answer will have an a and an h in it. Second, note that no calculus is required for this one, although you can find the answer using integrals and a bunch of work on line. It is always the case that the volume of a pyramid is

$$V = \frac{Ah}{3}$$

where A is the area of the base. The formula was known by Democritus b460 BCE using a method that is related to calculus but predating it by about 2000 years. If you want a reasonable explanation see this YouTube video, in particular see if you can find the hidden integral. Your job therefore is only to find the area of the base, i.e. the area of an equilateral triangle.

8. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis. $y = x^3, x = 0, x = 1, x = 2$

Cylindrical shells means we will use

$$V = \int_a^b 2\pi x f(x) dx$$

thinking that the circumference of a circle of radius x is $2\pi x$ and $f(x)$ is the height. I don't know why it is always written this way, since 2π is a constant it would be more normal to write

$$V = 2\pi \int_a^b x f(x) dx$$

in this case $a = 1, b = 2, f(x) = x^3$ so you only need to compute

$$2\pi \int_1^2 x^4 dx$$

a picture and the answers is Here

6.5 Average value of a function. Definition: If f is integrable on the interval $[a, b]$ then the average value is

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f$$

These problems are mostly and exercise in computing integrals, because there is not much else except dividing by the length of the path.

9. Find the average value of $f(x) = 8x - x^2$ over the interval $[0, 5]$

$$\frac{1}{5}\int_0^5 \left(8x - x^2\right) dx = \frac{1}{5}\left[4x^2 - \frac{x^3}{3}\right]\Bigg|_0^5 = \frac{1}{5} \times \frac{175}{3} = \frac{35}{3}$$

Notice we could have computed the integral and then divided by 5.

10. The mean value theorem for integrals says that the function must take on its average value somewhere in the interval. To find that number for the above function, set

$$8x - x^2 = \frac{35}{3}$$

solve the quadratic equation to get $x = 4 - \sqrt{\frac{13}{3}}$

11. Find the average value of $f(x) = 1 + \sqrt{x}$ on the interval $[1, 4]$

This time compute first, then divide by 3

$$\int_1^4 \left(1 + \sqrt{x}\right) dx = \int_1^4 dx + \int_1^4 x^{\frac{1}{2}} dx$$

The integral on the left is 3, the one on the right is

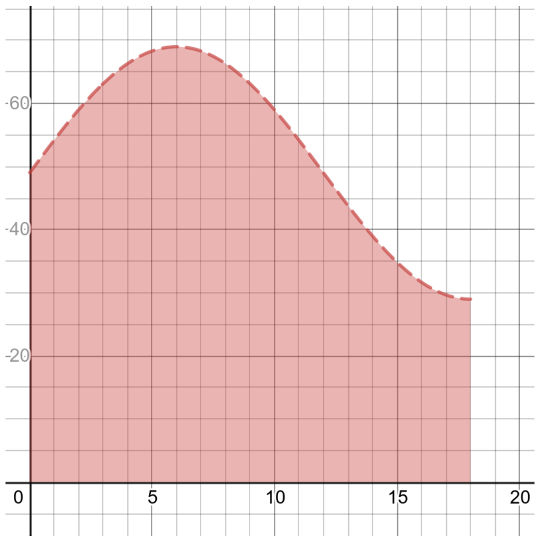
$$\frac{2}{3}\sqrt{x^3}\Bigg|_1^4 = \frac{2}{3}(8-1) = \frac{14}{3}$$

finally

$$3 + \frac{14}{3} = \frac{13}{3}, \frac{1}{3} \times \frac{13}{3} = \frac{23}{9}$$

12. The temperature at time t hours after 9 a.m. in a city was modeled by the function

$$F(t) = 49 + 20\sin\left(\frac{\pi t}{12}\right)$$



What was the average temperature between 9 a.m. and 3 a.m. The graph is the model
We need

$$\frac{1}{18}\int_0^{18}\left(49 + 20\sin\left(\frac{\pi t}{12}\right)\right) dt$$

$$\int_0^{18} 49dt = 18 \times 49 = 882$$

The anti derivative of $20\sin\left(\frac{\pi t}{12}\right)$ is

$$-\frac{240}{\pi}\cos\left(\frac{\pi t}{12}\right)$$

by a mental u-sub

$$\int_0^{18} 20\sin\left(\frac{\pi t}{12}\right) dt = -\frac{240}{\pi}\cos\left(\frac{\pi t}{12}\right)\Big|_0^{18} = -\frac{240}{\pi}(-1) = \frac{240}{\pi}$$

Answer

$$\frac{1}{18}\left(882 + \frac{240}{\pi}\right) \approx 53.2^\circ$$