1. Graph the parametric equation

$$x = t^3 - 3t, y = t^2 - 3$$

Before using wolfram lets plot some points, picking integer values of t from -5 to 5

t	$x = t^3 - 3t$	$y = t^2 - 3$	point
-5	-110	22	$(-110, 22)$
-4	-52	13	$(-52, 13)$
-3	-18	6	$(-18, 6)$
-2	-2	1	$(-2, 1)$
-1	2	-2	$(2, -2)$
0	0	-3	$(0, -3)$
1	-2	-2	$(-2, -2)$
2	2	1	$(2, 1)$
3	18	6	$(18, 6)$
4	52	13	$(52, 13)$
5	110	22	$(110, 22)$

Some things to notice:

$x = t^3 - 3t$ is an odd cubic function of t which means in the graph x will go from $-\infty$ to ∞

$y = t^2 - 3$ is an even quadratic function with minimum value -3 so y will go from -3 to ∞

The graph will not be a graph of a function because for example $(2, -2)$ and $(2, 1)$ are on the graph.

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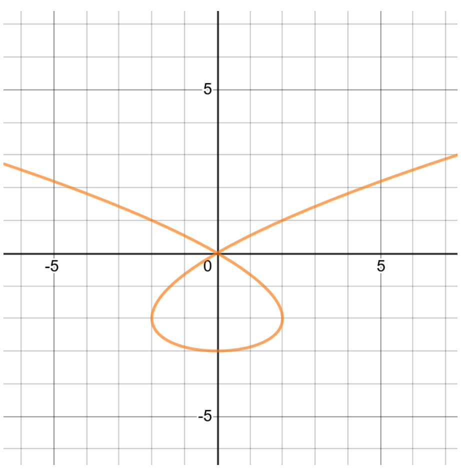

We can see from the graph that it contains the point $(0, 0)$ but we did not plot that point. To find the t that gives $(0, 0)$ which we will need to compute the slope of the tangent line at that point, we can solve

$$t^2 - 3 = 0 \iff t = \pm\sqrt{3}$$

We can check that if $y = \sqrt{3}$ then $x = \sqrt{3}^3 - 3\sqrt{3} = 0$

2. Graph

$$x = t^2 - 9, y = t + 3$$

Unlike the previous one we can "eliminate the parameter" by solving

$$y = t + 3 \iff y - 3 = t$$

and so

$$x = t^2 - 9 = (y - 3)^2 - 9$$

a parabola that opens to the right with vertex $(-9, 3)$

Note that when we eliminate the parameter we get a perfectly good equation, but without the parameter the graph does not have a direction.

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3. The mother of all parametric equations is given by

$$x = \cos(t), y = \sin(t)$$

i.e. the unit circle. If we take $0 \leq t < 2\pi$ the circle is traced out counter clockwise starting at $(0, 0)$. We could restrict t for say $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$ to get this:

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