Things to know from Calc 1

1. The Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$,then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$. Here you are cheated, because a proof is not given. You need to know something about real numbers that is discussed breifly in Chapter 11, namely the "Completeness Axiom".

2. The Mean Value Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b) then there is a number c in (a, b) with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

   We get a lot of mileage out of this theorem. The proof is in the book but you are still cheated because it relies on the Extreme Value Theorem.

3. If $f'(x) = 0$ for all x in (a, b) then f is constant on (a, b) .

   The proof is a straightforward application of the Mean Value Theorem. For any two numbers x_1, x_2 in (a, b) we have $f(x_1) - f(x_2) = 0$

4. If $f'(x) = g'(x)$ for all x then $f(x) = g(x) + C$ for some constant C.
   In English, if two functions have the same derivative they differ by a constant. The proof is an application of the theorem above, since $(f - g)' = 0$

5. (a) $\int_a^b f$ is a number
   (b) $\int_a^x f(t)dt$ is a function of x
   (c) $\int f$ are all functions F whose derivative is f
   (d) $\int_a^b f = -\int_b^a f$
   (e) $\int_a^a f = 0$
   (f) $\int_a^b f = \int_a^c f + \int_c^b f$

6. **The Fundamental Theorem of Calculus**

Suppose f is integrable on $[a, b]$ and $a \leq x \leq b$ If we put

$$F(x) = \int_a^x f(t)dt$$

then

$$F'(x) = f(x)$$

In English"the derivative of the integral is the integrand".

The proof is in the book, and very short, but it assumes you remember a bunch of stuff. The applications are important.

First however easy question

1. Find the derivative of

$$F(x) = \int_a^x \frac{t}{t^2+1}dt$$

If you understand what the theorem says there is no work

$$F'(x) = \frac{x}{x^2+1}$$

2. Find the derivative of

$$F(x) = \int_1^x e^{\cos(t)} \tan(t)\sqrt{t+1}dt$$

answer

$$F'(x) = e^{\cos(x)} \tan(x)\sqrt{x+1}$$

Is it always that easy? Sort of

3. Find the derivative of

$$F(x) = \int_x^5 \frac{t}{t^2+1}dt$$

First rewrite as

$$F(x) = -\int_5^x \frac{t}{t^2+1}dt$$

then it is that easy.

4. Find the derivative of

$$F(x) = \int_1^{\sin(x)} \frac{t}{t-2}dt$$

Think of this as a composite function (because it is) and use the chain rule.

$$F'(x) = \frac{\sin(x)}{\sin(x) - 2} \cos(x)$$

What happened? Replace t by $\sin(x)$ and then multiply by cosine because of the chain rule.

5. Evaluate

$$\int_{-2}^{3} \left(x^3 - 4x\right) dx$$

Question what happened to the t

Answer this is a number the variable is unimportant. The book uses x you could use ξ or whatever

$$\int_{-2}^{3} \left(\xi^3 - 4\xi\right) d\xi$$

Think of another function with the same derivative as

$$F(x) = \int_{-2}^{x} (t^3 - 4t)dt$$

I come up with

$$G(x) = \frac{x^4}{4} - 2x^2$$

Since the derivatives are the same by the FT of C, we know they can only differ by a constant.

What is the constant? We know

$$F(-2) = \int_{-2}^{-2} (t^3 - 4t)dt = 0$$

and $G(-2) = \frac{(-2)^4}{4} - 2 \times (-2)^2 = -4$ so the constant must be 4 and

$$\int_{-2}^{3} (x^3 - 4x)dx = \frac{3^4}{4} - 2 \times 3^2 + 4$$

Fundamental Theorem of Calculus Part II

If f is integrable and F is any anti derivative of f then

$$\int_{a}^{b} f = F(b) - F(a)$$

You will spend much of calc 2 finding anti-derivatives.