Worksheet on sigma notation.

1. Write out the sum $\sum_{k=1}^{5} 2k$

2. Use the distributive law to show $\sum_{k=1}^{5} 2k = 2\sum_{k=1}^{5} k$

3. Find

   (a) $\sum_{k=1}^{5} 1$

   (b) $\sum_{k=1}^{n} 1$

   (c) $\sum_{k=1}^{c} c$ where c is any constant.

4. Write out the sum $\sum_{k=1}^{5} k^2 + k$

5. Use the commutative law to show $\sum_{k=1}^{5} k^2 + k = \sum_{k=1}^{5} k^2 + \sum_{k=1}^{5} k$

6. Express in sigma notation: $1 - 3 + 5 - 7 + 9 - 11 + 13 - 15 + 17$

7. Observe that $1 = 1^2, 1 + 3 = 2^2, 1 + 3 + 5 = 3^2, 1 + 3 + 5 + 7 = 4^2$. What is the next sum?

8. Express this relationship in sigma notation.

9. Show that $\sum_{k=1}^{n} k^2 - (k-1)^2 = n^2$. Do not be confused by this: the n is the upper limit. Write out the first few terms without computing and you will see that the sum"telescopes".

10. Show that $k^2 - (k-1)^2 = 2k - 1$. This has nothing to do with sums, this is elementary algebra.

11. Conclude that $\sum_{k=1}^{n} 2k - 1 = n^2$ Not much work here, just put in the equal sign.

12. Using the above, show that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ by the following method: Rewrite $\sum_{k=1}^{n} 2k - 1 = n^2$ as $2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = n^2$ and solve for $\sum_{k=1}^{n} k$

13. Use the following formulas

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

to compute

(a) $\sum_{k=1}^{20} k(k-2)$

(b) $\sum_{k=1}^{10} k(k+1)(k+2)$

14. Compute $\sum_{k=1}^{n} \frac{k}{n^2}$. Do not get confused by the index k and the upper limit n which is a constant.

This is a harder set of problems analagous to problems 9 - 12:

15. Show that $\sum_{k=1}^{n} k^3 - (k-1)^3 = n^3$

16. Using elementary algebra, show that $k^3 - (k-1)^3 = 3k^2 - 3k + 1$

17. Conclude that $\sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3$

18. Sovle the above equation for $\sum_{k=1}^{n} k^2$ to arrive at the formula given above.

19. Express in sigma notation, and compute the number $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

20. Compute the number $\sum_{k=1}^{5} \frac{1}{2^k}$

21. Compute $\sum_{k=1}^{6} \frac{1}{2^k}$

22. What do you guess $\sum_{k=1}^{7} \frac{1}{2^k}$ will be?

23. Assumming we had a good definition for an infinite sum, what do you suppose

$$\sum_{k=1}^{\infty} \frac{1}{2^k}$$

should be?