171 test 3 Solutions

1. On which intervals is the function $f(x) = 2x^3 - 3x^2 - 36x$ increasing? Where is it decreasing? Here is a picture from maple to help, but you must show your work.

$$f'(x) = 6x^2 - 6x - 36 = 6\left(x^2 - x - 6\right) = 6(x-3)(x+2)$$

Since the derivative is a quadratic with positive leading coefficient, it is facing up. The zeros are –2 and 3, so it is positive outside the zeros and negative between them.

Answer: Increasing over $(-\infty, -2), (3, \infty)$, decreasing over $(-2, 3)$

2. Over what interval(s) is the above function concave up (leaning to the left) and concave down?

$f''(x) = 12x - 6$, a line, positive for $x > ½$, negative for $x < ½$.

Answer: concave up over $(1/2, \infty)$, concave down over $(-\infty, 1/2)$.

3. Locate the maximum and minimum values of the function $f(x) = 4\sqrt{x} - x^2$ on the interval $[0, 2]$. Don't forget to check the endpoints as well as the critical points.

$$f'(x) = \frac{2}{\sqrt{x}} - 2x \qquad \text{Set } \frac{2}{\sqrt{x}} - 2x = 0 \Rightarrow \frac{1}{\sqrt{x}} = x \Rightarrow 1 = x^{\frac{3}{2}} \Rightarrow x = 1$$

Critical point are $x = 0$ where the derivative is undefined, and $x = 1$.

$f(0) = 0, \qquad f(1) = 3, \qquad f(2) = 4\sqrt{2} - 4$

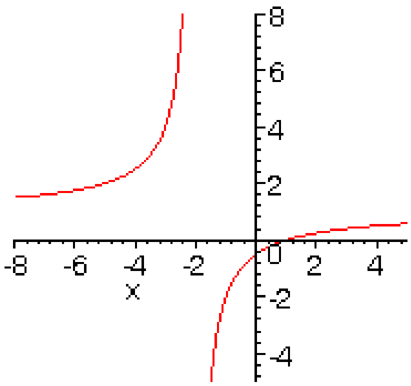
Answer: Minimum is 0, maximum is 3.

Let $f(x) = \dfrac{x-1}{x+2}$

4. f has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 1$.

5. Find $f'(x) = \dfrac{(x+2)-(x-1)}{(x+2)^2} = \dfrac{3}{(x+2)^2}$ and therefore always positive, hence f is increasing.

6. Using the information from 4 and 5 above, draw a rough sketch of f.



7. Approximate the root of $f(x) = x^3 - 2x - 2$ by using Newton's method. Start with the first guess of 2 and fill in the table.

x	y	y'
2	2	10
1.8		ignore

$$f(2) = 2^3 - 2\times 2 - 2 = 2, \qquad f'(2) = 3\times 2^2 - 2 = 10 \Rightarrow x_2 = 2 - \frac{2}{10} = 1.8$$

8. What does the mean value theorem say about the function $f(x) = x^3 - 2x - 2$ on the interval [0, 2]?
There is a number c between 0 and 2 with

$$f'(c) = 3c^2 - 2 = \frac{f(2) - f(0)}{2 - 0} = \frac{2 - (-2)}{2} = 2$$

9. For question above, find the number in the interval (0, 2) guaranteed by the mean value theorem to exist.

$$3c^2 - 2 = 2 \Rightarrow 3c^2 = 4 \Rightarrow c = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

Use L'Hôpital's rule if applicable to find the following limits:

10. $\lim_{x\to 0} \frac{\sin^2 x}{x^2}$

$\frac{\sin^2 0}{0^2} = \frac{0}{0}$ so L'Hôpital's rule is applicable.

$$\lim_{x\to 0} \frac{\sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin x \cos x}{2x} = \lim_{x\to 0} \frac{\sin 2x}{2x} = \lim_{x\to 0} \frac{2\cos x}{2} = \frac{2\cos 0}{2} = 1$$

or

$$\lim_{x\to 0}\frac{\sin^2 x}{x^2}=\lim_{x\to 0}\frac{2\sin x\cos x}{2x}=\lim_{x\to 0}\frac{\sin x\cos x}{x}=\lim_{x\to 0}\frac{\cos x\cos x-\sin x\sin x}{1}=\cos^2 0-\sin^2 0=1$$

11. $\lim_{x\to 3}\frac{\sqrt{x}-9}{x-3}$

$\frac{\sqrt{3}-9}{3-3}=\frac{\sqrt{3}-9}{0}\neq\frac{0}{0}$ so L'Hôpital's rule is not applicable. The limit does not exist.

12. $\lim_{x\to 0}\frac{e^{3x}-1}{x}$

$\frac{e^{3\times 0}-1}{0}=\frac{1-1}{0}=\frac{0}{0}$ so L'Hôpital's rule is applicable.

$$\lim_{x\to 0}\frac{e^{3x}-1}{x}=\lim_{x\to 0}\frac{3e^{3x}}{1}=3e^{3\times 0}=3e^{0}=3$$

13. Definition: If f is continuous on $[a , b]$, the **definite integral of *f* from *a* to *b* is**

$\int_a^b f(x)dx=$

Look it up.

14. Define each symbol on the right hand side of the equal sign above.

15. Suppose $\int_a^b f(x)dx=4$, $\int_a^c f(x)dx=-3$. What is $\int_c^b f(x)dx$?

$$\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f\left(x\right)dx\Rightarrow\int_c^b f\left(x\right)dx=7$$

16. For the function above, what is $\int_b^a f(x)dx$?

$$\int_b^a f(x)dx=-\int_a^b f\left(x\right)dx\Rightarrow\int_b^a f(x)dx=-4$$

17. Let $F(x)=\int_a^x f(t)dt$. F is a function of what variable? x

Which is why it is written F of x!!

18. For the function defined above, what is $F'(x)$?

$f(x)$

19. More specifically, if $F(x)=\int_1^x \sqrt{t^2-1}dt$ then $F'(x)=$

$f(x)=\sqrt{x^2-1}$

20. Evaluate $\int_1^4 \frac{1}{t^2}dt$

$f(t)=\frac{1}{t^2}=t^{-2} \Rightarrow F(t)=\frac{t^{-1}}{-1}=-\frac{1}{t}$

$F(4)-F(1)=-\frac{1}{4}-\left(-\frac{1}{1}\right)=1-\frac{1}{4}=\frac{3}{4}$

21. What are $\int_1^4 \frac{1}{z^2}dz$ and $\int_1^4 \frac{1}{x^2}dx$? Answer: $\frac{3}{4}$ and $\frac{3}{4}$

22. Let $F(x)=xe^x-e^x$ show that $F'(x)=xe^x$ (Don't forget the product rule!)

23. Evaluate $\int_0^1 xe^x dx$

$\int_0^1 xe^x dx=F(1)-F(0)=1e^1-e^1-\left(0e^0-e^0\right)=e-e+1=1$

Note that this works since the derivative of F is xe^x