Math 171 Test 2 Solutions

Find the following derivatives. Simplify if possible, but don't do anything silly.

2. $\frac{d}{dx}\left[\sqrt{\sin x}\right]$

Chain Rule: $\frac{1}{2\sqrt{\sin x}} \times \cos x = \frac{\cos x}{2\sqrt{\sin x}}$

3. $\frac{d}{dx}\left[\cos(2x)\right]$ Chain Rule again: $-\sin(2x) \times 2 = -2\sin(2x)$

4. $\frac{d}{dx}\left[e^{-x^2}\right]$ Another Chain Rule: $e^{-x^2} \times -2x = -2xe^{-x^2}$

5. $\frac{d}{dx}\left[\ln x \sin x\right]$ Produce Rule: $\frac{1}{x} \times \sin x + \ln x \times \cos x = \frac{\sin x}{x} + \ln x \cos x$

6. $\frac{d}{dx}\left[\frac{x+2}{x-2}\right]$ Quotient Rule: $\frac{(x-2)\times 1-(x+2)\times 1}{(x-2)^2} = \frac{x-2-x-2}{(x-2)^2} = \frac{-4}{(x-2)^2}$

7. $\frac{d}{dx}\left[x^{\sin x}\right]$ Logarithmic Differentiation:

$x^{\sin x} \times \frac{d}{dx}\left[\sin x \ln x\right] = x^{\sin x} \times \left(\frac{\sin x}{x} + \ln x \cos x\right)$

Note that this was the answer to question #5.

8. $\frac{d}{dx}\left[\frac{\sin x+2}{\sin x-2}\right]$ Same as Question #4 with x replaced by sin$x$. Therefore, by the Chain Rule and the answer to #4: $\frac{-4}{(\sin x-2)^2} \times \cos x = \frac{-4\cos x}{(\sin x-2)^2}$

9. Find the first and second derivatives of the function $f(x) = x\sin x$.

Product Rule: $f'(x) = 1\times \sin x + x\cos x = \sin x + x\cos x$

Product Rule again: $f''(x) = \cos x + 1\times \cos x + x \times -\sin x = -x\sin x + 2\cos x$

10. Explain in clear English why the derivative of the function $f(x)=10^x$ is not $x\times 10^{x-1}$ (i.e. why the power rule does not apply here.)
Because the variable is in the exponent!

11. What is the derivative of $f(x)=10^x$?
Logarithmic differentiation or memory:

$$f'(x)=10^x\times\frac{d}{dx}[x\ln 10]=10^x\times\ln 10$$

12. What is the derivative of $\log_{10}x$?

$$\frac{1}{x\ln 10}$$

13. Find the linear approximation to the function $f(x)=\dfrac{1}{x-1}$ at $a = 2$

$$f(2)=\frac{1}{2-1}=1,\ f'(2)=-\frac{1}{(2-1)^2}=-1\Rightarrow L(x)=1+-1(x-2)=1-x+2=3-x$$

14. Find the slope of the line tangent to the curve $\sqrt{x}+\sqrt{y}=5$ at the point (4,9).

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}y'=0\Rightarrow\frac{1}{2\sqrt{y}}y'=-\frac{1}{2\sqrt{x}}\Rightarrow y'=-\frac{\sqrt{y}}{\sqrt{x}}.\qquad x=4,y=9,y'=-\frac{\sqrt{9}}{\sqrt{4}}=-\frac{3}{2}$$

15. Use the fact that $\dfrac{d}{dx}\left[\tan^{-1}(x)\right]=\dfrac{1}{1+x^2}$ and the chain rule to find $\dfrac{d}{dx}\left[\tan^{-1}(e^x)\right]$.

$$\frac{1}{1+\left(e^x\right)^2}\times e^x=\frac{e^x}{1+e^{2x}}$$

16. Let $f(x)=\sin x, g(x)=\ln x, h(x)=\sqrt{x}$

$$f'(x)=\cos x, g'=\frac{1}{x}, h'(x)=\frac{1}{2\sqrt{x}}$$

$$f\circ g\circ h(x)=\sin\ln\sqrt{x}$$

17. Use the chain rule to find the derivative of $f\circ g\circ h(x)$

$$\cos\left(\ln\sqrt{x}\right)\times\frac{1}{\sqrt{x}}\times\frac{1}{2\sqrt{x}}=\frac{\cos\left(\ln\sqrt{x}\right)}{2x}$$

18. For *f*, *g* and *h* above, find the derivative of their product. That is, find

$$\frac{d}{dx}\left[\sin x \cdot \ln x \cdot \sqrt{x}\right]$$

There are many ways to proceed. Since we have seen the derivative of $\sin x \ln x$ twice before, we can use that as one piece for the product rule. Or, use

$$(fgh)' = f'gh + fg'h + fgh'$$

First way: $\left(\frac{\sin x}{x} + \ln x \cos x\right)\sqrt{x} + (\sin x \ln x)\frac{1}{2\sqrt{x}} = \frac{\sin x\sqrt{x}}{x} + \ln x \cos x\sqrt{x} + \frac{\sin x \ln x}{2\sqrt{x}}$

19. Suppose $z = \ln y$, and $\frac{dy}{dx} = 6x$. What is $\frac{dz}{dx}$? (Your answer will have an *x* and a *y* in it.)

$$\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx} = \frac{1}{y} \times 6x = \frac{6x}{y}$$

20. Suppose $f^2(t) + g^2(t) = 169$ and $g'(t) = 3$. What is $f'(t)$ when $g(t) = 5$?

$2ff' + 2gg' = 0 \Rightarrow f' = \frac{-gg'}{f}$ Put $g = 5, g' = 3, f = 12$ or -12 $\left(\text{since } 12^2 + 5^2 = 169\right)$

Get $f' = \pm\frac{15}{12}$