

1. Definition: The **tangent line** to the curve $y = f(x)$ at the point $(a, f(a))$ is the line with slope m : =

2. Definition: the **instantaneous rate of change** of the function $f(x)$ at $(a, f(a))$ is

3. Definition: The **derivative of a function f at a number a** , denoted by $f'(a)$, is

4. For example, if the function is the exponential function e^x , then the derivative at a is

5. Another example is the cosine function. Its derivative at a is

6. State, as precisely as you can either in clear English or using mathematics, what it means to say $\lim_{x \rightarrow a} f(x) = L$

7. Find: $\lim_{x \rightarrow 3} \frac{x^2 - 3x - 10}{x + 2}$

8. Find $\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x + 2}$

9. Definition: a **rational function** is

10. An example of a rational function is

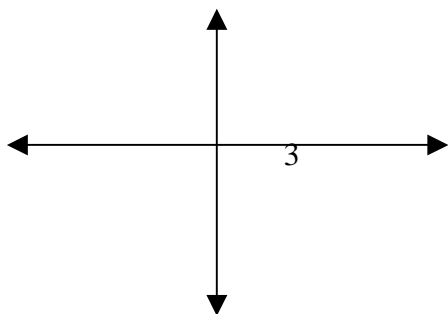
11. What is the domain of the function you defined in question 10?

12. Where is the function you defined in question 10 continuous?

13. Definition: a function is **continuous** at a point a in its domain if

14. Definition: a function is **continuous on an interval** if

15. Draw a picture of a function that is *not* continuous at $x=3$



What kind of discontinuity did you draw?

16. How would you define $f(x) = \frac{x^2 - 3x - 10}{x + 2}$ at -2 so that it would be continuous there (and hence everywhere)?

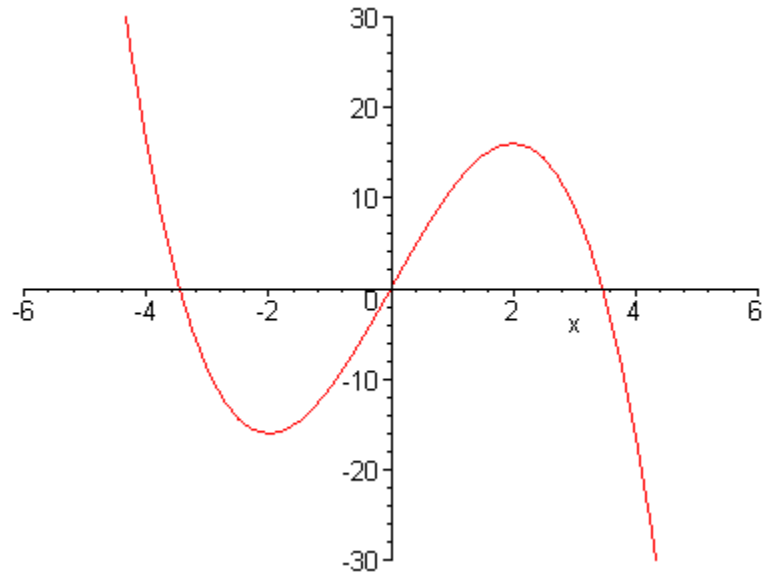
17. Use the definition of the derivative to find the formula for the slope of the tangent line to the graph of $y = x^2 + 3x - 5$ at a point $(a, a^2 + 3a - 5)$. (Answer: $2a + 3$)

18. Find the equation of the line tangent to the graph of $y = x^2 + 3x - 5$ at $(1, -1)$.

19. What is the vertex of the parabola in problem 16?

20. Let $f(x) = x^3 - 2x^2 + 3x - 5$. $f(1) = -3$, $f(2) = 1$. Why must f have a root in the interval $(1, 2)$?

Here is the picture you expected: Let's say this is the graph of $y = f(x)$



21. For what values of x is $f'(x) = 0$?
22. Is $f'(-4)$ positive or negative?
23. Is $f'(0)$ positive or negative?
24. More generally, for which values of x is $f'(x)$ positive and for which values is it negative? (Your answer should be either inequalities or interval notation.)

25. The function above is $f(x) = -x^3 + 12x$. Use the definition to find $f'(x)$, and confirm your answers in numbers 22 through 25.