

1. Write the converse, inverse and contrapositive of the compound statement: If pigs could fly there would be bacon in the trees.

(a) converse

(b) inverse

(c) contrapositive

2. Write a truth table for  $(p \vee q) \rightarrow (p \wedge q)$

$p$	$q$	$p \vee q$	$p \wedge q$	$(p \wedge q) \rightarrow (p \vee q)$
$T$	$T$			
$T$	$F$			
$F$	$T$			
$F$	$F$			

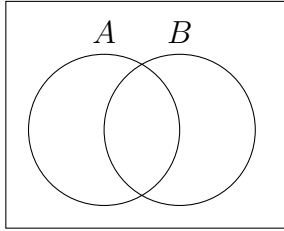
3. Prove  $\neg(p \wedge q) \equiv \neg p \vee \neg q$  using a truth table.

4. Negate the statement "Every good boy does fine."

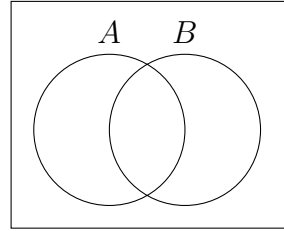
5. Negate the statement  $\forall xP(x)$

6. Show  $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$  using Venn diagrams.

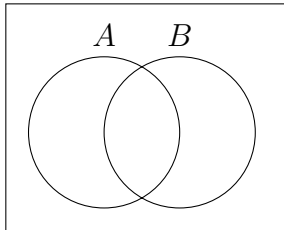
(a)  $A \cup B$



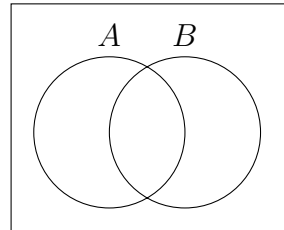
(d)  $\overline{B}$



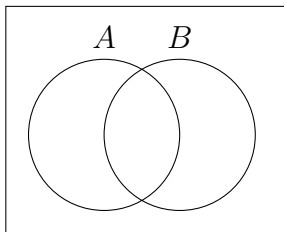
(b)  $A \cap B$



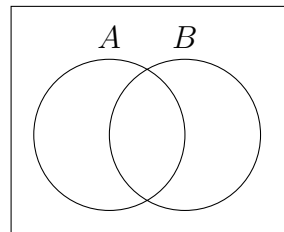
(e)  $\overline{A} \cap \overline{B}$



(c)  $\overline{A}$



(f)  $\overline{(A \cup B)}$



Write the following statements in English, and negate each statement (in English). Determine whether the original statement is true, and give a justification. The domain of each statement is  $\mathbb{N} = \{1, 2, 3, \dots\}$

7.  $\forall x \exists y (y = 2x)$

(a) English

(b) Negation

(c) True or False

(d) Justification

8.  $\forall y \exists x (y = 2x)$

(a) English

(b) Negation

(c) True or False

(d) Justification

9. Definition: A function  $f : A \rightarrow B$  is injective (one to one) if and only if

10. Negate the definition:  $f : A \rightarrow B$  is not injective if and only if

11. Definition: A function  $f : A \rightarrow B$  is surjective (onto) if and only if

12. Negate the definition:  $f : A \rightarrow B$  is not surjective if and only if

13. Let  $f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{0\}$  via  $x \mapsto \frac{1}{x-1}$  i.e.  $f(x) = \frac{1}{x-1}$

(a) Show that  $f$  is injective.

(b) Find an explicit formula for  $f^{-1}(x)$

14. Prove the (rather obvious) fact that if  $n$  is an integer and  $3n - 2$  is even, then  $n$  is even

(a) by contraposition

(b) by contradiction