

Show that each of the following statements is a tautology

1.  $(p \wedge q) \rightarrow (p \rightarrow q)$

$p$	$q$	$p \wedge q$	$p \rightarrow q$	$p \wedge q \rightarrow (p \rightarrow q)$
$T$	$T$			
$T$	$F$			
$F$	$T$			
$F$	$F$			

2.  $\neg(p \rightarrow q) \rightarrow \neg q$

$p$	$q$	$\neg q$	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg q$

3.  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$p$	$q$	$r$	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

4. Show each of the above is a tautology without using truth tables.

5. Show that  $p \rightarrow q$  and  $\neg q \rightarrow \neg p$  are logically equivalent.

6. Show that  $p \leftrightarrow q$  and  $\neg(p \oplus q)$  are logically equivalent.

7. Show that  $p \leftrightarrow q$  and  $(p \rightarrow q) \wedge (q \rightarrow p)$  are logically equivalent.

8. Show that  $(p \rightarrow q) \vee (p \rightarrow r)$  and  $p \rightarrow (q \vee r)$  are logically equivalent.