

Newton's Law of Cooling states that the temperature of a heated object *decreases exponentially* over time toward the temperature of the surrounding medium. That is, the temperature u of a heated object at a given time t satisfies the equation

$$u = T + (u_0 - T)e^{kt}$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object and k is a negative number.

Example: An object is heated to 100 degrees C and is then allowed to cool in a room whose air temperature is 30 degrees C. If the temperature of the object is 80 degrees after 5 minutes, when will its temperature be 50 degrees?

Solution: $T = 30, u_0 = 100$ so $u = 30 + (100 - 30)e^{kt} = 30 + 70e^{kt}$

We know when $t = 5, u = 80$, so we solve for k in the equation

$$80 = 30 + 70e^{k5}$$

$$50 = 70e^{k5}$$

$$\frac{5}{7} = e^{k5}$$

$$\ln \frac{5}{7} = 5k$$

$$k = \frac{1}{5} \ln \frac{5}{7} \approx -0.0673$$

Therefore the formula becomes $u = 30 + 70e^{-0.0673t}$

1. Now set $u = 50$ and solve for t .

You put a nice warm 72-degree bottle of beer in a refrigerator where the temperature is a constant 38 degrees F. After 2 minutes the beer is 60 degrees.

1. What will the temperature be after 7 minutes?
2. How long will it take before the temperature reaches 39 degrees?

You take a pot of boiling oatmeal off the stove at 8:00 and its temperature is 212 degrees. At 8:05 the temperature is 200 degrees.

3. In the absence of any other information, what would you expect the temperature to be at 8:10?
4. Suppose we also know that the room temperature is 70 degrees. What do you know the temperature will be at 8:10?