Week 3 Notes

1. Let's solve the exponential equation

$$5e^x = 35$$

(a) First, we isolate e^x to get the equivalent equation

$$e^x = 7$$

(b) Next, write in equivalent logarithmic form

$$x = \ln(7)$$

(c) Now we use a calculator to find $x =$ wolfram (Round your answer to three decimal places.)

2. Let's solve the logarithmic equation

$$\log(4) + \log(x - 2) = \log(x)$$

(a) First, we combine the logarithms on the LHS to get the equivalent equation

$$\log(4x - 8) = \log(x)$$

(b) Next, we use the fact that log is one-to-one to get the equivalent equation

$$4x - 8 = x$$

(c) Now we find

$$x = \frac{8}{3}$$

3. Find the solution of the exponential equation, as in Example 1

$$5^{2x-4} = 1$$

$$5^{2x-4} = 1 \iff 2x - 4 = 0 \iff x = 2$$

4. Consider the following.

$$e^{-8x} = 17$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

$$e^{-8x} = 17 \iff -8x = \ln(17) \iff x = -\frac{\ln(17)}{8}$$

(b) Use a calculator to find an approximation to the solution rounded to six decimal places. $x =$ wolfram

5. Consider the following.

$$4e^{.02x} = 2$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

$$4e^{.02x} = 2 \iff e^{.02x} = .5 \iff .02x = \ln(.5) \iff x = \frac{\ln(.5)}{.02} = -50\ln(2)$$

(b) Use a calculator to find an approximation to the solution rounded to six decimal places. $x =$ wolfram

6. Consider the following.

$$200\,(1.025)^{5t} = 1100$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

$$200\,(1.025)^{5t} = 1100 \iff (1.025)^{5t} = 5.5 \iff 5t = \frac{\log(5.5)}{\log(1.025)} = \iff t = \frac{\log(5.5)}{5\log(1.025)}$$

(b) Use a calculator to find an approximation to the solution rounded to six decimal places. t = wolfram

7. Solve

$$5^{x+4} = 7^x$$

First take the log of both sides to get

$$\log(5^{x+4}) = \log(7^x)$$

Then use the property of the logs $\log(b^x) = x\log(b)$
to put the exponents as multipliers and write

$$(x+4)\log(5) = x\log(7)$$

Distribute on the left to get

$$x\log(5) + 4\log(5) = x\log(7)$$

Subtract xlog(5) from both sides:

$$4\log(5) = x\log(7) - x\log(5)$$

Factor an x out on the right hand side to get

$$4\log(5) = x\,(\log(7) - \log(5))$$

And finally divide to to solve for x:

$$\frac{4\log(5)}{\log(7) - \log(5)} = x$$

8. Consider the following.

$$\frac{12}{1+e^{-x}} = 2$$

(a) Find the exact solution of the exponential equation in terms of logarithms.

$$\frac{12}{1+e^{-x}} = 2 \iff 1+e^{-x} = 6 \iff e^{-x} = 5 \iff -x = \ln(5) \iff x = -\ln(5)$$

(b) Use a calculator to find an approximation to the solution rounded to six decimal places. x = diy

9. Solve the equation. (Enter your answers as a comma-separated list. Round your answers to four decimal places.)

$$2x^2e^{3x} + 5xe^{3x} - e^{3x} = 0$$

The gimmick here is that e^{whatever} is never 0 so this is just

$$2x^2 + 5x - 1 = 0$$

a quadratic equation, use the quadratic formula, or cheat wolfram

10. Solve the logarithmic equation for x, as in Example 7. (Enter your answers as a comma-separated list.)

$$\log(x) + \log(x-2) = \log(24)$$

the base is unimportant here because the left and right sides are both logs. First combine, then get rid of the logs, then solve

$$\log(x)+\log(x-2) = \log(24) = \log(x^2-2x) = \log(24) \iff x^2-2x = 24 \iff x^2-2x-24 = 0$$

this one factors

$$x^2 - 2x - 24 = 0 \iff (x-4)(x+6) = 0 \iff x = 4 \text{ or } x = -6$$

but we have to be careful here. While 4 or -6 are solutions to the quadratic, they are not solutions to the logarithmic equation, because -6 is not in the domain of the log $(0, \infty)$ so the only solution is $x = 4$

11. Solve the logarithmic equation for x. (Enter your answers as a comma-separated list.)

$$\log_2(x) + \log_2(x-3) = 2$$

This one the base is important, because the right hand side is a number.

$$\log_2(x) + \log_2(x-3) = 2 \iff \log_2(x^2 - 3x) = 2 \iff x^2 - 3x = 2^2$$

Now your job is to solve $x^2 - 3x - 4 = 0$ and check that your solutions are in the domain of the log.

12. A man invests \$2000 in an account that pays 7.5% interest per year, compounded quarterly. (a) Find the amount after 3 years? (Round your answer to the nearest cent.)

$$2000\left(1 + \frac{0.075}{4}\right)^{12} = \ldots$$

(b) How long will it take for the investment to double? (Round your answer to two decimal places.)

The formula is

$$2000\left(1 + \frac{0.075}{4}\right)^{4t}$$

When it doubles, there will be \$4000 so we could be dumb and write

$$2000\left(1 + \frac{0.075}{4}\right)^{4t} = 4000$$

and solve for t but the reason this is silly is that step one will be to divide by 2000 and get

$$\left(1 + \frac{0.075}{4}\right)^{4t} = 2$$

so the initial value is unimportant, doubling time is doubling time.

$$\left(1 + \frac{0.075}{4}\right)^{4t} = 2 \iff 4t = \frac{\ln(2)}{\ln(1 + \frac{0.075}{4})} \iff t = \frac{\ln(2)}{4\ln(1 + \frac{0.075}{4})}$$

$t =$ wolfram