Quadratic Equations

1. Solve the following equations:

   (a) $x^2 = 1$

   (b) $x^2 = 9$

   (c) $x^2 = 16$

   (d) $x^2 = 25$

   (e) $x^2 = \frac{49}{4}$

   (f) $x^2 = \frac{25}{9}$

2. Use the above to solve the following:

   (a) $(x+4)^2 = 1$

   (b) $(x-5)^2 = 9$

   (c) $(x+4)^2 = 16$

   (d) $(x-1)^2 = 25$

   (e) $(x+\frac{1}{2})^2 = \frac{49}{4}$

   (f) $(x-\frac{5}{6})^2 = \frac{25}{9}$

3. Solve the following equations, write your answer in simplest radical form:

   (a) $x^2 = 12$

   (b) $x^2 = 50$

   (c) $x^2 = 45$

   (d) $x^2 = \frac{5}{4}$

   (e) $x^2 = \frac{28}{4}$

   (f) $x^2 = \frac{3}{4}$

4. Use the above to solve the following:

   (a) $(x+3)^2 = 12$

   (b) $(x-5)^2 = 50$

   (c) $(x+1)^2 = 45$

   (d) $(x+\frac{3}{2})^2 = \frac{5}{4}$

   (e) $(x-\frac{1}{2})^2 = \frac{28}{4}$

   (f) $(x-\frac{5}{2})^2 = \frac{3}{4}$

To make $x^2 + bx$ a "perfect square" add $\left(\frac{b}{2}\right)^2$ to obtain $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

This allows for a simple method of solving a quadratic equation. Add the appropriate number to both sides to write the left hand side as a perfect square, then procede as in the previous examples.

Example 1:

$x^2 - 10x + 16 = 0$

$x^2 - 10x = -16$

$(x - 5)^2 = -16 + 25$

$(x - 5)^2 = 9$

Here $b = -10, \frac{b}{2} = -5, (-5)^2 = 25$ We obtain a perfect square by adding 25.

Example 2:

$x^2 + 3x + 1 = 0$

$x^2 + 3x = -1$

$\left(x + \frac{3}{2}\right)^2 = -1 + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{5}{4}$

In this example $b = 3, \frac{b}{2} = \frac{3}{2}, \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Note that it is easier to work with fractions than decimals.

Example 3:

$2x^2 - 10x + 11 = 0$

$2x^2 - 10 = -11$

$x^2 - 5x = -\frac{11}{2}$

$\left(x - \frac{5}{2}\right)^2 = -\frac{11}{2} + \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{3}{4}$

In this example we needed to divide by 2 to make the leading coefficient 1.

Solve the following quadratic equations as in the following example:

$x^2 + 8x + 3 = 0$

$x^2 + 8x = -3$

$(x + 4)^2 = -3 + 16$

$(x + 4)^2 = 13$

$x + 4 = \sqrt{13}$ or $x + 4 = -\sqrt{13}$

$x = -4 + \sqrt{13}$ or $x = -4 - \sqrt{13}$

1. $x^2 - 6x + 3 = 0$
2. $x^2 + 7x + 2 = 0$
3. $x^2 + x - 1 = 0$
4. $2x^2 - 6x - 5 = 0$
5. $2x^2 + 31x - 51 = 0$
6. $\frac{2}{3}x^2 - x + \frac{1}{3} = 0$ Hint: mulitply by 3.
7. $4x^2 + 12x + 9 = 0$
8. $3x = 2(x^2 - 1)$

$$ax^2 + bx + c = 0$$