Week 9 Description
11.3
11.4
11.5
The Integral Test Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)dx$ is convergent.

In other words:

If $\int_1^{\infty} f(x)dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.
If $\int_1^{\infty} f(x)dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.
We don't want to use this test, there are easier things to do than compute an integral, but it is very useful for the following result: We know $\int_1^{\infty} \frac{1}{x^p}$ converges if $p > 1$ diverges if $p \leq 1$ so know we know

$$\sum \frac{1}{n^p} \text{converges} \iff p > 1$$

We combine this with 11.4 Comparison test to get quick answers to whether a series converges or not.
The limit Comparison Test
Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.
Most frequently we compare to a p series

1. $\sum \frac{\log(n)}{n}$ diverges because $\frac{\log(n)}{n} > \frac{1}{n}$ and we know $\sum \frac{1}{n}$ diverges. Here we actually use the direct comparison test.

2. $\sum \frac{n}{\sqrt{n^5-2}}$ will easily be seen to converge by the comparison test, but we have to know **Compared to What**

   Ignore the -2 which contributes nothing , get

$$\frac{n}{\sqrt{n^5}} = \frac{n}{n^{\frac{5}{2}}} = \frac{1}{n^{\frac{3}{2}}}$$

   and that is what you compare to. The limit of the ratios will be 1, and we know $\sum \frac{1}{n^{\frac{3}{2}}}$ converges because $\frac{3}{2} > 1$

   11.5

   Alternating series test say if the terms alternate in sign, in other words if $b_n > 0$ and we have

$$\sum (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 + \dots$$

   then the series will converge if $\{b_n\}$ is decreasing and $\lim_{n\to\infty} b_n = 0$

3. The harmonic series $\sum \frac{1}{n}$ diverges, but the alternating harmonic series $\sum \frac{(-1)^n}{n}$ converges.