1. Graph the parametric equation

$$x = t^3 - 3t, y = t^2 - 3$$

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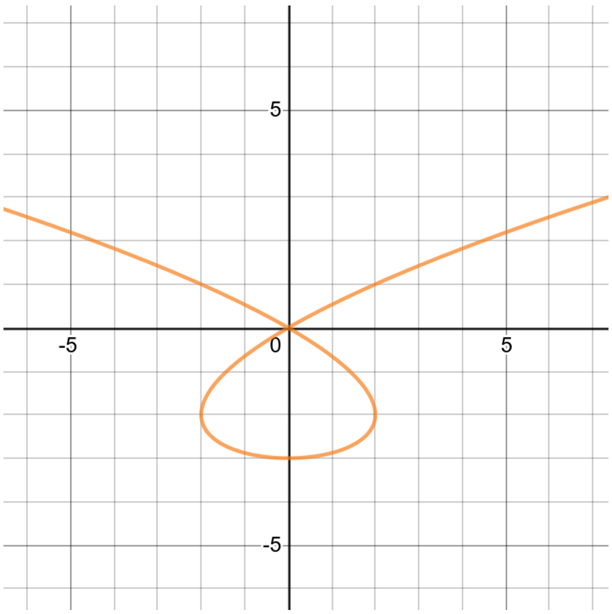


2. Find the slope of the line tangent to the curve at the point $(2, 1)$ In order to do this we need

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2 - 3}$$

Take a second to note how easy this is compared to almost anything we have done so far, especially series.

The only non-trivial part is finding $t$. Since $y = 1$ we have

$$t^2 - 3 = 1 \iff t^2 = 4 \iff t = \pm 2$$

so there are two possibilities for $t$. To see which one to pick, put $t = -2$ in $x = t^3 - 3t$ get $x = (-2)^3 - 3(-2) = -2$ which is not what we want, but does tell us that at $t = -2$ we have the point $(-2, 1)$ on the graph.

$t = 2$ gives $x = 2$. Therefore the slope is

$$m = \frac{2 \times 2}{3 \times 2^2 - 3} = \frac{4}{9}$$

Q why didn't we start with $t^3 - 3t = 2$?

A because it is a cubic equation.

3. Find the slope of the line tangent to the curve at $(0,0)$

   We can see from the graph that it contains the point $(0,0)$ but we did not plot that point. To find the $t$ that gives $(0,0)$ which we will need to compute the slope of the tangent line at that point, we can solve

   $$t^2 - 3 = 0 \iff t = \pm\sqrt{3}$$

   At $t = -\sqrt{3}$ we get $-\dfrac{1}{\sqrt{3}}$ and

   at $t = \sqrt{3}$ we get $\dfrac{1}{\sqrt{3}}$

   We can see from the graph that there are two tangent lines at $(0,0)$

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4. Find the points on the graph where the tangent line is vertical.

   $$\frac{dy}{dx} = \frac{2t}{3t^2 - 3}$$

   is undefined if $3t^2 - 3 = 0 \iff t = \pm 1$

   At $t = -1, x = 2, y = -2$ point is $(2,-2)$ and at $t = 1$ we get $(-2,-2)$

5. Find the point where the tangent line is horizontal. DIY

6. Find the area of the enclosed loop in the above parametric equation.

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   $$\int_\alpha^\beta y(t)x'(t)dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^2 - 3)(3t^2 - 3)dt = \frac{24\sqrt{3}}{5}$$

   Wolfram

7. Find the length of that the curve enclosing the loop.

   $$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{(3t^2 - 3)^2 + (2t)^2} dt$$

   Wolfram