Week 12 Description
10.1
10.2
Parametric Equations = Use wolfram. I cannot visualize these, maybe you can. Some are simple enough, as in **this video**
Others are much harder

1. Graph $x = \sin(5t), y = 1 + \cos(3t)$ We know that as t goes from 0 to 2π, $\sin(5t)$ goes from -1 to 1 ten times, and $1 + \cos(3t)$ will go from 0 to 2 six times.

   Together they look like this **Wolf**

   Use Wolfram to do this homework, it will be easy.

   For 10.2 There are three main ideas: First:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

2. For example if $x = \cos(t), y = t^2 + 1$ **Wolf**

   then

$$\frac{dy}{dx} = -\frac{2t}{\sin(t)}$$

   Nothing so hard about that. Tangent will be vertical when denominator is zero, (and the numerator is not) so

$$t = \pi, x = -1, y = \pi^2 + 1$$

   Second, Area from $t = \alpha$ to $t = \beta$ is

$$\int_{\alpha}^{\beta} y(t)x'(t)dt$$

3. For example the area enclosed by the ellipse $x = \cos(t), y = 2\sin(t)$ **Wolf** is

$$\int_{2\pi}^{0} 2\sin(t) \times (-\sin(t))dt = -2\int_{2\pi}^{0} \sin^2(t)dt = 2\pi$$

Third, the length of the curve from $t = \alpha$ to $t = \beta$ is

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Amazing part about this is that they make up problems you can actually do, which usually means the thing inside the radical is a perfect square. Try for example

4. $x = e^t - t, y = 4e^{\frac{t}{2}}$ for $0 \leq t \leq 2$

   But we don't care it the integral is easy or not. For example

5. If $x = \cos(t), y = t^2 + 1$ for $0 \leq t \leq 4$ then the arc length is given by the integral

$$\int_0^4 \sqrt{\sin^2(t) + 4t^2} dt$$

   Wolf