The Fundamental Theorem of Calculus

Suppose f is integrable on $[a, b]$ and $a \leq x \leq b$ If we put

$$F(x) = \int_a^x f(t)dt$$

then

$$F'(x) = f(x)$$

In English "the derivative of the integral is the integrand".
The proof is in the book, and very short, but it assumes you remember a bunch of stuff. The applications are important. First some very easy questions:

1. Find the derivative of

   $$F(x) = \int_a^x \frac{t}{t^2+1}dt$$

   If you understand what the theorem says there is no work

   $$F'(x) = \frac{x}{x^2+1}$$

2. Find the derivative of

   $$F(x) = \int_1^x e^{\cos(t)} \tan(t)\sqrt{t+1}dt$$

   answer

   $$F'(x) = e^{\cos(x)} \tan(x)\sqrt{x+1}$$

   Is it always that easy? Sort of

3. Find the derivative of

   $$F(x) = \int_x^5 \frac{t}{t^2+1}dt$$

   First rewrite as

   $$F(x) = -\int_5^x \frac{t}{t^2+1}dt$$

   then it is that easy.

4. Find the derivative of

   $$F(x) = \int_1^{\sin(x)} \frac{t}{t-2}dt$$

   Think of this as a composite function (because it is) and use the chain rule.

   $$F'(x) = \frac{\sin(x)}{\sin(x)-2}\cos(x)$$

   What happened? Replace t by $\sin(x)$ and then multiply by cosine because of the chain rule.

5. Evaluate

$$\int_{-2}^{3} \left(x^3 - 4x\right) dx$$

Question: What happened to the t?

Answer: This is a number, the variable is unimportant. The book uses x you could use ξ or whatever

$$\int_{-2}^{3} \left(\xi^3 - 4\xi\right) d\xi$$

Think of another function with the same derivative as

$$F(x) = \int_{-2}^{x} (t^3 - 4t)dt$$

I come up with

$$G(x) = \frac{x^4}{4} - 2x^2$$

Since the derivatives are the same by the FT of C, we know they can only differ by a constant.

What is the constant? We know

$$F(-2) = \int_{-2}^{-2} (t^3 - 4t)dt = 0$$

and $G(-2) = \frac{(-2)^4}{4} - 2 \times (-2)^2 = -4$ so the constant must be 4 and

$$\int_{-2}^{3} (x^3 - 4x)dx = \frac{3^4}{4} - 2 \times 3^2 + 4$$

Fundamental Theorem of Calculus Part II

If f is integrable and F is any anti derivative of f then

$$\int_{a}^{b} f = F(b) - F(a)$$

You will spend much of calc 2 finding anti derivatives, even though wolfram can do it without you learning anything.