Examples of u-subs you do in your head:

1. $$\int \cos(5x)dx = \frac{1}{5}\sin(5x) + C$$

   the mental substitution was $u = 5x$

2. $$\int e^{\frac{x}{2}}dx = 2e^{\frac{x}{2}} + C$$

   via $u = \frac{x}{2}$

3. $$\int \frac{dx}{3x} = \frac{1}{3}\ln(x) + C$$

   this is not a substitution because $\frac{1}{3x} = \frac{1}{3} \times \frac{1}{x}$

4. $$\int \sec^2(\pi x)dx = \frac{1}{\pi}\tan(\pi x) + C$$

5. $$\int x^3 \sin(x^4)dx = -\frac{1}{4}\cos(x^4) + C$$

   via $u = x^4$

6. $$\int xe^{3x^2}dx = \frac{1}{6}e^{3x^2} + C$$

   via $u = 3x^2$

Examples of substitutions that you may need to write

1.
$$\int \frac{x}{\sqrt{1-x^2}}dx$$
the substitution is $u = 1 - x^2, du = -2xdx, -\frac{du}{2} = dx$
$$\int \frac{x}{\sqrt{1-x^2}}dx = -\frac{1}{2}\int \frac{1}{\sqrt{u}}du = -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}2u^{1/2} = -\sqrt{1-x^2} + C$$

2.
$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx$$
the substitution is $u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}, 2du = \frac{dx}{\sqrt{x}}$
$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx = 2\int \cos(u)du = 2\sin(u) = 2\sin(\sqrt{x}) + C$$

3.
$$\int \tan(x)dx$$
need to recall that $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ making the substitution
$$u = \cos(x), du = -\sin(x)dx, -du = \sin(x)dx$$
$$\int \tan(x)dx = -\int \frac{du}{u} = -\ln(u) = -\ln(\cos(x)) = \ln(\sec(x)) + C$$
the last equality by the property of the log that says $-\ln(x) = \ln(\frac{1}{x})$ and also remembering that secant is the reciprocal of cosine.

Example of u-sub where du is not there.

$$\int \frac{1}{1+\sqrt{x}}$$

Note, if you look at wolfram's step by step answer there are two things that are not clear, or obvious. The natural inclination is to let $u = \sqrt{x}$ but it is easier to go to make the entire denominator u

$$u = 1 + \sqrt{x}$$

and solve for x

$$u = 1 + \sqrt{x}, u - 1 = \sqrt{x}, (u-1)^2 = x, 2(u-1)du = dx$$

then

$$\int \frac{1}{1+\sqrt{x}} = 2\int \frac{u-1}{u} du = 2\int 1 du - 2\int \frac{1}{u} du = 2u - 2\ln(u) = 2\sqrt{x} - 2\ln(1+\sqrt{x}) + C$$

it would actually be $2 + 2\sqrt{x} - 2\ln(1+\sqrt{x})$ but 2 is just a constant so we don't write it, it gets combined with the +C at the end.