172 Homework 8 Name: ______________________________

1. Definition (page 285 Spivak)
$$\log(x) =$$
For example:
   (a) $\log(2) =$
   (b) $\log(\pi) =$
   (c) $\log(1) =$

2. Definition (page 287) The exponential function
$$\exp(x) =$$

3. Definition (page 287)
$$e =$$

4. Definition (page 288)
$$e^x =$$

5. Explain in clear English (see above) why $\log(1) = 0$

6. Explain in clear English why that makes $\exp(0) = e^0 = 1$

7. Definition: for any $b > 0$ and any real number x
$$b^x =$$

For example:

(a) $2^{\pi} =$

(b) $2^{x} =$

(c) $x^{\frac{1}{x}} =$

(d) $(1-2x)^{\frac{1}{x}} =$

(e) $(\sin(x))^{x} =$

(f) $x^{\sin(x)} =$

8. Find the derivatives of the above.

9. By definition $2^x =$ and $2^y =$

10. Use the above to write $2^x \times 2^y$

11. By definition $2^{x+y} =$

12. Use the property of the exponential function $e^\alpha \times e^\beta = e^{\alpha+\beta}$ to prove they are equal, that is, to prove $2^x \times 2^y = 2^{x+y}$

13. By definition $2^{-1} =$

14. Use the definition to prove $2^{-1} = \frac{1}{2}$ hint: you will need the property of the logs that says $-\log(x) = \log(\frac{1}{x})$ Find the following limits

15. $\lim_{x \to 0} (1 - 2x)^{\frac{1}{x}}$

16. $\lim_{x \to \infty} x^{\frac{1}{x}}$

17. $\lim_{x \to 1^+} x^{\frac{1}{1-x}}$

18. $\lim_{x \to \infty} x^{\frac{\log(2)}{1-\log(x)}}$