172 Homework 6 Name: ______________________________

Integrating rational functions.

1. Simplest example:

$$\int \frac{x^3}{x-2}dx$$

   (a) Divide $\frac{x^3}{x-2}$ using synthetic division. You will get a quotient and remainder.

   (b) After division, the integral $\displaystyle\int \frac{x^3}{x-2}dx$ is easy.

2. Example

$$\frac{x-2}{x+1} = \frac{x+1-3}{x+1} = 1 - \frac{3}{x+1}$$

   And therefore

$$\int \frac{x-2}{x+1}dx = \int (1 - \frac{3}{x+1})dx = x - 3\ln(|x+1|) + C$$

   (a) $\displaystyle\int \frac{x+3}{x-2}dx$

   (b) $\displaystyle\int \frac{x-2}{x-4}dx$

(c) $\displaystyle\int \frac{2x+7}{x+2}dx$

(hint: the quotient will be 2, so force it by making part of the numerator $2x+4$

3. Partial fractions simplest example:

$$\frac{x-7}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \iff A(x+2) + B(x-1) = x-7 \text{ for all x}$$

(a) If $x = 1, A(1+2) = 1-7 \iff 3A = -6 \iff A = -2$

(b) If $x = -2, B(\underline{\qquad}) \iff \underline{\quad}B = \underline{\quad} \iff B = \underline{\quad}$

(c) Now the integral is easy enough.

$$\int \frac{x-7}{(x-1)(x+2)}dx =$$

4. Repeat the above to find the partial fraction decomposition of

$$\frac{7x-13}{(x-3)(x+1)}$$

Then find

$$\int \frac{7x-13}{(x-3)(x+1)}dx$$

5. Try doing this partial fractions the cowboy way

$$\frac{6}{(x-2)(x+1)} = \frac{\qquad}{x-2} + \frac{\qquad}{x+1}$$

Making $\displaystyle\int \frac{6}{(x-2)(x+1)} dx =$

6. $\displaystyle\int \frac{x^2+2x-1}{(x-1)(x^2+1)} dx$ hint, here the partial fractions will look like $\displaystyle\frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

7. An easy partial fractions gives $\displaystyle\frac{1}{x^2-a^2} = \frac{1}{2a(x-a)} - \frac{1}{2a(x+a)}$

   (a) Therefore, $\displaystyle\int \frac{dx}{x^2-a^2} =$

   (b) Complete the square and use the above to find
   $\displaystyle\int \frac{dx}{x^2+2x-4}$

8. Another easy partial fractions gives $\displaystyle\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

9. $\int \frac{dx}{1+e^x}$ in steps:

(a) Make the substitution $u = e^x$ solve for x get $x = \_\_$

(b) That makes $dx = \_\_du$

(c) Making the substitution gives $\int \frac{dx}{1+e^x} =$

Your answer here should be all in terms of u

(d) The resulting integral in u is already solved in question 8. It is

(e) Replace u in the above answer by e^x. Also remember that no one writes $\ln(e^x)$

10.

$$\int \frac{2x^2-6}{(x-1)(x+1)^2}dx$$