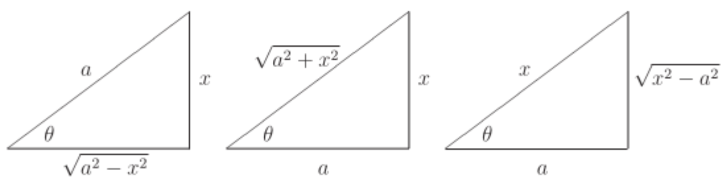
172 Homework 5 Name: ______________________________

If use see	use the sub	so that	and
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$dx = a\cos\theta\,d\theta$	$\sqrt{a^2-x^2} = a\cos\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$dx = a\sec^2\theta\,d\theta$	$\sqrt{a^2+x^2} = a\sec\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$dx = a\sec\theta\tan\theta\,d\theta$	$\sqrt{x^2-a^2} = a\tan\theta$

In each line, the last entry follows from the second entry by one of the Pythagorean identities.

There are also right triangles you can draw to make the connections between x, a, and θ. The three triangles below refer to the three trig subs, respectively.



1. $\displaystyle\int \frac{x^2dx}{\sqrt{9-x^2}}$

2. $\displaystyle\int \frac{dx}{\sqrt{16+x^2}}$

3. $\displaystyle\int \frac{\sqrt{x^2-25}}{x^3}dx$

It should be clear from question 2 that $\displaystyle\int \frac{dx}{\sqrt{u^2+a^2}} = \ln(|u+\sqrt{u^2+a^2}|$ Use the above result and the miracle of completing the square to instantly find

4. $\displaystyle\int \frac{dx}{\sqrt{x^2+4x+10}}$

5. $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x^3\sqrt{x^2-1}}$

Hint: this looks hard but if you change the limits of integration you don't have to go back to x. The substitution is $x = \sec(\theta)$ making $\theta = \sec^{-1}(x)$ So, for example, the lower limit is $\theta = \sec^{-1}(\sqrt{2}) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$