172 Homework 4 Name: ______________________________

Trigonmetric integrals

1. Simplest example:

$$\int \sin^2(x)\cos^3(x)dx$$

   (a) Rewrite $\cos^3(x)$ as $\cos^2(x)\cos(x) = (1-\sin^2(x))\cos(x)$

   (b) Rewrite the integral with the above.

   (c) Make is simple u-substitution $u = \sin(x)$ so $du =$__________

   (d) Compute the integral after the substitution.

   (e) Replace u by $\sin(x)$

2. $$\int \tan^3(x)dx$$

   Hint: rewrite $\tan^2(x) = \sec^2(x) - 1$ then make the substitution $u = \tan(x)$

There are three "double angle" formula for $\cos(2x)$

$$\cos(2x) = 2\cos^2(x) - 1$$
$$\cos(2x) = 1 - 2\sin^2(x)$$

and a third we don't need.

3.
$$\int \cos^2(x)dx$$

(a) Solve $\cos(2x) = 2\cos^2(x) - 1$ for $\cos^2(x)$ gives $\cos^2(x) = \frac{1}{2} + \frac{\cos(2x)}{2}$

(b) The mental u-sub $u = 2x, dx = \frac{1}{2}du$ gives

$$\int \cos^2(x)dx = \int \frac{1}{2} + \frac{\cos(2x)}{2}dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C = \frac{1}{2}\left(x + \cos(x)\sin(x)\right) + C$$

4. Repeat the above to find
$$\int \sin^2(x)dx$$

Two well known but hard to prove integrals are

$$\int \sec(x)dx = \ln(|\sec(x)\tan(x)| + C$$

and

$$\int \sec^3(x)dx = \frac{1}{2}\left(\sec(x)\tan(x) + \ln(|\sec(x) + \tan(x)|\right) + C$$

5. Use the above to compute

$$\int \tan^2(x)\sec(x)dx$$

6. Use the result from question 4, and integration by parts to compute

$$\int x\sin^2(x)dx$$