

1. Define what it means to say $\lim_{x \rightarrow a} f(x) = L$ in clear English

2. Definition: a function f is continuous at a if

3. Definition: a function f is continuous on an interval (a, b) if

4. What is the domain of the function $f(x) = \frac{x}{x^2 - 4x}$

5. Where is the function $f(x) = \frac{x}{x^2 - 4x}$ continuous?

6. Find $\lim_{x \rightarrow 2} \frac{x}{x^2 - 4x}$

7. Find $\lim_{x \rightarrow 0} \frac{x}{x^2 - 4x}$

8. If $\lim_{x \rightarrow a} f(x) = 4$, $\lim_{x \rightarrow a} g(x) = -1$ find $\lim_{x \rightarrow a} \frac{\sqrt{f(x)} - g(x)}{f(x)g(x)}$

9. How could you define $f(x) = \frac{x^2 - 2x - 8}{x - 4}$ so that it will be continuous at 4 (and therefore continuous everywhere)?

10. Find the following limits as $x \rightarrow \infty$ quickly and without any agony:

(a) $\lim_{x \rightarrow \infty} \frac{4x^2 + 3x + 10}{2x^2 + 2x - 7}$

(b) $\lim_{x \rightarrow \infty} \frac{x}{x^2 - 1}$

(c) $\lim_{x \rightarrow \infty} \frac{5x}{\sqrt{9x^2 + 4x}}$

11. Let

$$f(x) = \begin{cases} x + 3 & \text{if } x < 2 \\ x^2 - 1 & \text{if } x \geq 2 \end{cases}$$

(a) Find $\lim_{x \rightarrow 2^-} f(x)$

(b) Find $\lim_{x \rightarrow 2^+} f(x)$

(c) Is f continuous at 2?

12. Definition: the slope of the line tangent to the graph of $y = f(x)$ at $(a, f(x))$ is $m :=$

13. Use the definition to find the slope of the line tangent to the graph of $y = \frac{1}{x}$ at $(3, \frac{1}{3})$.
You should know what the answer is, but use the definition.

14. Definition: The derivative of a function $f(x)$ is the function $f'(x) =$

15. Using the definition, write (but do not try to compute) the derivative of the function $f(x) = \tan(x) - e^x + \frac{1}{x-1}$

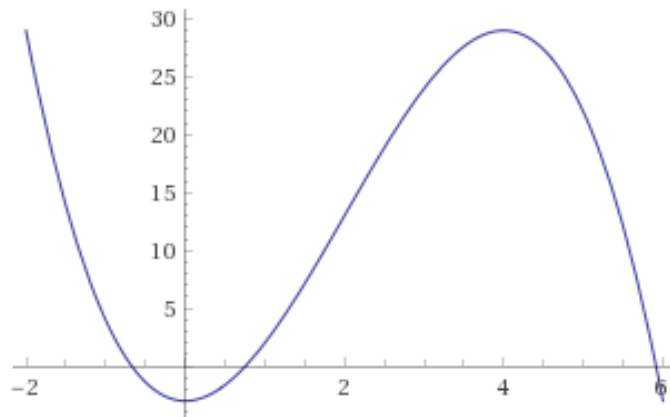
16. Definition: a function f is differentiable over the interval (a, b) if

17. Use the definition to find the derivative of $f(x) = x^2 - 2x - 4$
You should know the answer before you start

18. Find the slope of the line tangent to the graph of $y = x^2 - 2x - 4$ at $(3, -1)$ and find the equation of that line.

19. For which value of x is $f'(x) = 0$ for the above function?

Below is the graph of $y = f(x)$ for a cubic function f



Computed by Wolfram|Alpha

20. For which x is $f'(x) = 0$?

21. Is $f'(1)$ positive or negative?

22. Is $f'(-1)$ positive or negative?

23. More generally, for which x is $f'(x) > 0$? Your answer should be written either as inequalities or in interval notation.

24. Compute $\lim_{x \rightarrow \infty} \sqrt{x^2 + 2x} - x$

25. Give an example of a function that is continuous at $x = 2$ but not differentiable at $x = 2$. You can either use an equation to define the function or draw a picture.

