Week 4 notes

Some facts:

1. The slope of the line tangent to $y = f(x)$ at the point $(a, f(a))$ is
$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$
2. $f'(a)$ is called the "derivative" of f at a
3. If $f(x) = x^2$ then $f'(a) = 2a$
4. If $f(x) = x^3$ then $f'(a) = 3a^2$
5. If $f(x) = \sqrt{x}$ then $f'(a) = \dfrac{1}{2\sqrt{a}}$
6. If $f(x) = mx + b$ a line, then $f'(a) = m$ the slope of the line

1. Find the slope of the line tangent to $f(x) = 10x - x^2$ at $(2, 14)$
$$f'(a) = 10 - 2a \to f'(2) = 10 - 4 = 6$$
Find the equation of the line:
The slope is 6 and the point is $(2, 14)$ via point-slope formula
$$y - 14 = 6(x - 2)$$
solve for y
2. Find the equation of the line tangent to $y = \sqrt{x}$ at the point $(25, 5)$
The slope is $\dfrac{1}{2\sqrt{25}} = \dfrac{1}{10}$ use
$$y - 5 = \frac{1}{10}(x - 25)$$
3. Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 6$ if $g(6) = 7$ and $g'(6) = -2$
The point is $(6, 7)$ the slope is -2 use $y - 7 = -2(x - 6)$
4. $$f(t) = \frac{2t + 10}{t + 2}$$
$$f'(a) = \lim_{h\to 0} \frac{\dfrac{2(t+h) + 10}{t + h + 2} - \left(\dfrac{2t + 10}{t + 2}\right)}{h}$$
and a boat load of algebra.

Probably easier is

$$\lim_{t\to a} \frac{\frac{2t+10}{t+2} - \frac{2a+10}{a+2}}{t-a}$$

The numerator is

$$\frac{(2t+10)(a+2)-(t+2)(2a+10)}{(t+2)(a+2)} = \frac{6(a-t)}{(t+2)(a+2)}$$

Divide by $t-a$ i.e. cancel get

$$\lim_{t\to a} \frac{-6}{(t+2)(a+2)} = -\frac{6}{(a+2)^2}$$

5.

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h}$$

is the derivative of $f(x)=x^3$ at $a=2$ because by definition

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

and if $f(x)=x^3, a=2$ we get

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h}$$

6.

$$\lim_{x\to 0} \frac{\sin(x)}{x}$$

is the derivative of $\sin(x)$ at $x=0$ since we could write it as

$$\lim_{x\to 0} \frac{\sin(x)-\sin(0)}{x-0}$$