We will do the first two homework problems together. Then work on computing limits.

1. Find

$$\lim_{x\to 4} \frac{x^2+4x}{\sqrt{x+5}}$$

Nothing could be easier. Put in $x = 4$. If get a number, that is the answer.

2.

$$\lim_{x\to 5} \frac{x^2-25}{x-5}$$

If we put in 5 we get "literally" $\frac{0}{0}$ but that does not mean the limit is undefined, that means there is more work to do. In this case the work is easy

$$\frac{x^2-25}{x-5} = \frac{(x+5)(x-5)}{x-5} = x+5 \text{ if } x \neq 5$$

notice we write if $x \neq 5$ because you can't cancel zeros. But no matter, when we take the limit as x approaches 5, we are specifically saying x is close to but not equal to 5. Therefore

$$\lim_{x\to 5} \frac{x^2-25}{x-5} = \lim_{x\to 5} x+5 = 5+5 = 10$$

Now if you are thinking, you might reasonably ask what kind of an idiot would write $\frac{x^2-25}{x-5}$ when they really mean $x+5$? The answer is we are that kind of idiot. If we want to find the slope of the line tangent to the curve $y = x^2$ at the point $(5, 25)$ then computing the change in y over the change in x, we would get

$$\frac{x^2-25}{x-5}$$

and then let x get close to 5 i.e. take the limit, and get a slope of 10.

3.

$$\lim_{x\to 2} \frac{x^2+4x}{x^2-4}$$

This time we literally get $\frac{12}{0}$ no more work to do, does not exist.

4.

$$\lim_{x\to 7} \frac{x^2+5x-84}{x^2-5x-14}$$

Again we get $\frac{0}{0}$ so more work to do, namely factor and cancel.

$$\frac{x^2+5x-84}{x^2-5x-14} = \frac{(x-7)(x+12)}{(x-7)(x+2)} = \frac{x+12}{x+2} \text{ if } x \neq 7$$

so

$$\lim_{x\to 7} \frac{x^2+5x-84}{x^2-5x-14} = \lim_{x\to 7} \frac{x+12}{x+2} = \frac{19}{9}$$

5. What kind of trick is this? How come when we get $\frac{0}{0}$ for a rational function, we can always factor and cancel? No trick at all, the "factor theorem" (see math 161) says that if r is a zero of a polynomial $p(x)$ the p factors as $(x-r)q(x)$

6.
$$\lim_{h\to 0}\frac{(4+h)^3-64}{h}$$
Again $\frac{0}{0}$ and so the h has to factor out of the numerator. We just have to expand
$$(4+h)^3 = h^3+12h^2+48h+64$$
making
$$\frac{(h+4)^3-64}{h} = \frac{h^3+12h^2+48h}{h} = h^2+12x+48$$
and therefore
$$\lim_{h\to 0}\frac{(4+h)^3-64}{h} = \lim_{h\to 0} h^2+12x+48 = 48$$

7.
$$\lim_{t\to 4}\frac{4-t}{2-\sqrt{t}}$$
This time no polynomial but we have two ways of solving. One is to rationalize the denominator
$$\frac{4-t}{2-\sqrt{t}}\times\frac{2+\sqrt{t}}{2+\sqrt{t}} = \frac{(4-t)(2+\sqrt{t})}{4-t} = 2+\sqrt{t}$$
The other way, which is probably less obvious, is to factor the numerator
$$\frac{4-t}{2-\sqrt{t}} = \frac{(2-\sqrt{t})(2+\sqrt{t})}{2-\sqrt{t}} = 2+\sqrt{t}$$
either way we get
$$\lim_{t\to 4} 2+\sqrt{t} = 2+2 = 4$$

8.
$$\lim_{x\to 13}\frac{\sqrt{x+3}-4}{x-13}$$
same as the previous one, rationalize the numerator and the $x-13$ will cancel.

9.

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

I give this question to 161 students on test 1, but without the limit, just to get rid of the compound fraction. There are two ways to do it, multiply top and bottom by $3x$ to get

$$\frac{3 - x}{3x(x-3)} = -\frac{1}{3x}$$

or to do the subtraction in the numerator

$$\frac{1}{x} - \frac{1}{3} = \frac{3 - x}{3x}$$

and then divide by $x - 3$ to get the same answer

$$-\frac{1}{3x}$$

Therefore

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} = \lim_{x \to 3} -\frac{1}{3x} = -\frac{1}{9}$$

10.

$$\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$$

Don't make up your own algebra: $x^{-1} = \frac{1}{x}$ so this is

$$\lim_{h \to 0} \frac{\frac{1}{(3+h)} - \frac{1}{3}}{h}$$

Do the subtraction in the numerator

$$\frac{1}{3+h} - \frac{1}{3} = \frac{3 - (3+h)}{3(3+h)} = -\frac{h}{3(3+h)}$$

then divide by h i.e. cancel, then put in $h = 3$

11. A function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

We will do the last problems together.