Notes on logs;

$$\log_b(x) = y \iff b^y = x$$

$$\log(x) = y \iff 10^y = x$$

$$\ln(x) = y \iff e^y = x$$

Laws of Logarithms If x and y are positive numbers, then

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a\left(x^r\right) = r\log_a x$ (where r is any real number)

   1. $$\log_2(\sqrt{8}) = \frac{3}{2} \iff 2^{\frac{3}{2}} = 8$$
   2. $$\ln(e^3 = 3 \iff e^3 = e^3$$
   3. $$\log(a) + \frac{1}{3}\log(b)) = \log(a) + \log(\sqrt[3]{b}) = \log(a\sqrt[3]{b})$$
   4. All mighty change of base formula (so we can evaluate any log)

      $$\log_b(x) = \frac{\ln(x)}{\ln(b)} = \frac{\log(x)}{\log(b)}$$

      (a) $$\log_7(100) = \frac{\log(100)}{\log(7)}$$

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      (b) $$7^x = 200 \iff x = \log_7(200) = \frac{\ln(200)}{\ln(7)}$$
   5. $$e^{.2x+4} = 20 \iff .2x + 4 = \ln(20) \iff .2x = \ln(20) - 4 \iff x = \frac{\ln(20) - 4}{.2}$$
   6. $$\ln(2 - 3x) = 4 \iff 2 - 3x = e^4 \iff -3x = e^4 - 2 \iff x = -\frac{e^4 - 2}{3}$$

7. Find the domain of $f(x) = \ln(2 - \ln(x))$
   The domain of $\ln(x)$ is $x > 0$ so our job is to solve

$$2 - \ln(x) > 0 \iff \ln(x) < 2 \iff x < e^2$$

   making the solution $0 < x < e^2$

8. for the function above, find $f^{-1}(x)$

   Put $x = \ln(2 - \ln(y)$ and solve for y

$$x = \ln(2 - \ln(y)) \iff e^x = 2 - \ln(y) \iff \ln(y) = 2 - e^x \iff y = e^{2-e^x}$$

9. Inverse trig Functions: just make sure the range is correct and compositions:
   (a) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ not $\frac{11\pi}{6}$
   (b) $\sin(\cos^{-1}(x)) = \sqrt{1 - x^2}$ because of the mother of all trig identities $\sin^2(\theta) + \cos^2(\theta) = 1$
   (c) $\sin(\tan^{-1}(x))$
   (d) $\tan(\cos^{-1}(x))$
   (e) $\sin(2\tan^{-1}(x))$
   For the last four draw a triangle. The last one requires remembering that $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$