MATH 171 TEST 1 NAME________________________________

1. Definition: $\lim_{x\to a} f(x) = L$ means

We can force *f* to be as close to *L* as we like by making *x* close to (but not equal to) *a*

2. Definition: a function *f* is continuous at a number *a* if

$\lim_{x\to a} f(x) = f(a)$

i.e. the limiting value and the function agree at that point.

3. What is the domain of the function $f(x) = \frac{x^2+6x}{2x^2-x}$?

$2x^2 - x = x(2x-1) \neq 0 \Rightarrow x \neq 0, x \neq \frac{1}{2}$

4. Definition: a function is **continuous on an interval** if

It is continuous at every number in the interval.

5. Where is the function $f(x) = \frac{x^2+6x}{2x^2-x}$ continuous?

Everywhere except 0, 1/2.

6. What is $\lim_{x\to 2} \frac{x^2+6x}{2x^2-x}$?

$$\frac{2^2+6\times 2}{2\times 2^2-2} = \frac{4+12}{8-2} = \frac{16}{6} = \frac{8}{3}$$

7. What is $\lim_{x\to\infty} \frac{x^2+6x}{2x^2-x}$

Degree of numerator = degree of denominator so limit at infinity is ratio of the leading coefficients = 1/2

8. What is $\lim_{x\to 0} \frac{x^2+6x}{2x^2-x}$

$$\frac{x^2+6x}{2x^2-x}=\frac{x(x+6)}{x(2x-1)}=\frac{x+6}{2x-1},\ x\neq 0\Rightarrow \lim_{x\to 0}\frac{x^2+6x}{2x^2-x}=\lim_{x\to 0}\frac{x+6}{2x-1}=\frac{6}{-1}=-6$$

9. Definition: The **tangent line** to the curve $y=f(x)$ at the point $(a, f(a))$ is the line with slope m: =

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

10. Definition: The **derivative of a function *f* at a number** *a*, denoted by $f'(a)$, is

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

For 11 and 12 you are not expected to compute, just write what you would need to compute to find the derivatives.

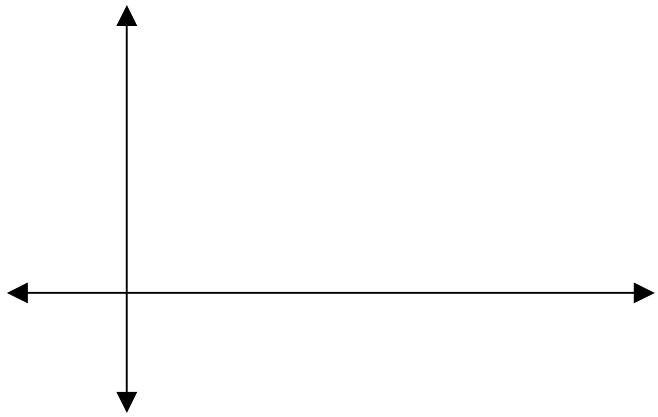
11. If the function is the logarithmic function ln(*x*) then the derivative at *a* is

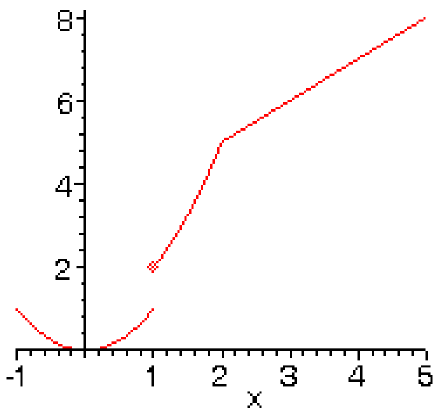
$$\lim_{h\to 0}\frac{\ln(a+h)-\ln(a)}{h}$$

12. Another example is the function $f(x)=\frac{\sqrt{x}}{1-x}$. Its derivative at *a* is

$$\lim_{h\to 0}\frac{\frac{\sqrt{a+h}}{1-(a+h)}-\frac{\sqrt{a}}{1-a}}{h}$$

13. Draw a picture of a function with a removable discontinuity at *x* = 2





For the function *f* defined by the graph above, find the following:

14. $\lim\limits_{x \to 1^+} f(x) = 2$

15. $\lim\limits_{x \to 1^-} f(x) = 1$

16. $\lim\limits_{x \to 1} f(x) =$ does not exist

17. $f(1) = 2$

18. Explain in clear English or in mathematics why *f* is continuous at 2.

Because the limit of the function and the value of the function agree at $x = 2$ i.e. $\lim\limits_{x \to 2} f(x) = f(2) = 5$

19. Is *f* differentiable at 2? Why, or why not.

NO, because *f* has a "corner" there. If you said Yes because it is continuous be advised that continuity does not imply differentiability.

For problems 20 – 24 let $f\left(x\right)=\dfrac{x^2+3x-10}{x+5}$

20. What is the domain of *f*?

All real numbers except –5

21. Where is *f* continuous?

All real numbers except –5

22. Find $\lim\limits_{x\to -5}\dfrac{x^2+3x-10}{x+5}$

$$\lim_{x\to -5}\frac{x^2+3x-10}{x+5}=\lim_{x\to -5}x-2=-7$$

23. What kind of discontinuity does *f* have at x = -5?

Removable.

24. How would you define *f* at x = -5 so that it was continuous there?

At x = -5 define *f* to be –7, i.e. $f\left(-5\right)=-7$

Now the discontinuity is removed, hence the term *removable*.

25. Use the definition of the derivative (not the power rule) to find the formula for the slope of the tangent line to the graph of $y=x^2-2x+6$ at a point (a, a^2-2a+6). Just grind it out; it is not that hard. (Answer: $2a-2$)

$$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + 6 - (x^2 - 2x + 6)}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h + 6 - x^2 + 2x - 6}{h}$$

$$= \lim_{h \to 0} \frac{2xh - 2h + h^2}{h} = \lim_{h \to 0} 2x - 2 + h = 2x - 2$$

Or, if you use *a* instead of *x* you get the same expression only with an *a* in place of *x*.

26. Find the slope of the line tangent to the graph of $y = x^2 - 2x + 6$ at (2, 6).

$m = f'(2) = 2 \times 2 - 2 = 2$

27. Now that you have the slope, use the point slope formula to find the equation of the line tangent to the graph at (2,6)

$y - 6 = 2(x - 2)$ or $y = 2x + 2$

28. What is the vertex of the parabola above? $(1, 5)$

Either use the formula for the vertex $\left(\frac{-b}{2a}, \text{ something} \right)$ or else first answer the question below, then set $2a - 2 = 0$ and solve for *a* to get *a* = 1.

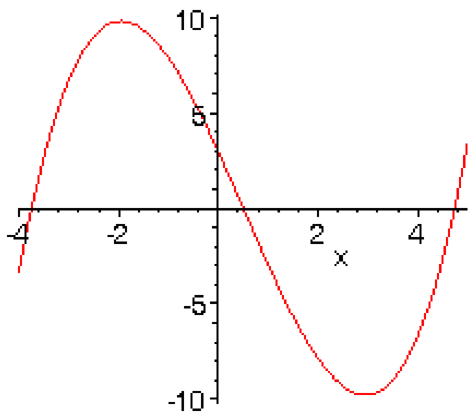
29. What is the slope of the line tangent to the graph of the parabola at the vertex?

ZERO!!

Either use the formula above to get the vertex, then substitute 1 in the formula for the slope and get 0, or else note that at the vertex the tangent line must be horizontal, and therefore the slope must be zero.

30. Let $g(x) = \frac{1}{1-x}$ Clearly $g(0) = 1, g(2) = -1$ Must g have a zero between x = 0 and x = 2? Why or why not.

NO. In fact g is never zero. For a fraction to be zero the numerator must be zero, but the numerator here is 1. Perhaps you view this as a trick, but you must check the hypothesis of a theorem before you can apply it. The Intermediate value theorem does not apply because g is not continuous on the interval [0,2]. g has a discontinuity at $x = 1$



This is the graph of $y = f(x)$

For what values of x is $f'(x) = 0$?

31. Is $f'(-4)$ positive or negative? It is positive because the function is headed up.

32. Is $f'(0)$ positive or negative?

Negative. (Going down!)

33. More generally, for which values of x is $f'(x)$ positive and for which values is it negative? (Your answer should be either inequalities or interval notation.)

The function is headed up until about –2, then down until about 2.5 or 3, then up again. A reasonable guess would be something like positive on the intervals $(-4,-2),(3,5)$ and negative in between. If you assumed the picture is just a piece of the function and wrote $(-\infty,-2),(3,\infty)$ that is fine too, although it really does say it **is** the graph of the function, not a piece of it, and therefore the domain is only what you see.