

Test 3

1. How many 5 letter “words” can be formed from the letters in the word “whiskey”?
2. How many 8 letter “words” can be formed from the letter in the word “calculus”?
3. Write the first 8 levels of Pascal’s triangle.
4. Use the 5th level to expand $(a + b)^5$
5. Find the coefficient of the term x^7 in the expansion of $(x - 2)^{12}$
6. Show that if there are 30 students in a class, at least two must have last names that begin with the same letter.
7. Show that among any group of 6 integers, there is a pair x, y with $x \equiv y \pmod{5}$
8. How many cards must be drawn from a deck to ensure that there are either 4 clubs, 3 hearts, 2 spades or 1 diamond selected?
9. Verify Pascal’s identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

for $n = 6, k = 4$

10. Let A be the set $\{1, 2, 3, 4, 5\}$ and R be the relation $R = \{(x, y) | x \text{ divides } y\}$
 - (a) Write R as a set.
 - (b) Draw a diagram for R
 - (c) Is R reflexive? Be explicit.
 - (d) Is R symmetric?
 - (e) Transitive?
11. The sets $\{a, b\}, \{c, d\}, \{e\}$ form a partition of $S = \{a, b, c, d, e\}$ Diagram the corresponding equivalence relation.
12. Let $A = \{0, 1, 2, 3, 4, 5, 6\}$ and R be the relation $R = \{(x, y) | x \equiv y \pmod{3}\}$
 - (a) Show explicitly that R is an equivalence relation.
 - (b) What are the equivalence classes of R ?

13. Add $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

14. Add $\sum_{k=100}^{200} (4k - 5)$

15. Add $\sum_{k=1}^{50} k^3 + k$

16. Add $\sum_{k=2}^{\infty} \frac{3}{(k-1)(k+2)}$

17. Prove the well ordered principle is logically equivalent to strong induction.