163 Test 2 Name: ____________________

1. Find the first 5 terms of the sequence defined by the recurrence relation

$$a_n = a_{n-1} + n, a_1 = 1$$

2. Show that $a_n = 2^n - 1$ is a solution to the recurrence relation

$$a_n = 2a_{n-1} + 1,$$

3. Find $\sum_{k=1}^{100} 2k + 5$

4. Find $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k$

5. Prove $\sum_{j=1}^{n} 2j + 5 = n^2 + 6n$ by induction

   (a) Base step:

   (b) Induction hypothesis:

   (c) Inductive step:

6. Prove that $n^2 + n$ is even by induction:

   (a) Base step:

   (b) Induction hypothesis:

   (c) Inductive step:

7. Prove $\forall n \in \mathbb{N}, 5|11^n - 6^n$ that is $11^n - 6^n$ is divisible by 5

   (a) Base step:

   (b) Induction hypothesis:

   (c) Inductive step: Hint $11^{k+1} - 6^{k+1} = 11^{k+1} - 11 \times 6^k + 5 \times 6^k$

8. Factor 392 and 420

9. Find $lcm(420, 392)$

10. It is fairly clear that $gcd(101, 23) = 1$ since both are prime. Using the Euclidean algorithm find the Bezout coefficents, that is, find s and t so that $101s + 23t = 1$

11. What is 101 modulo 23?

12. Find the inverse of 4 modulo 9

13. Solve $4x \equiv 3(\mod 9)$

14. Solve the system

$$x \equiv (2 \bmod 3), x \equiv (1 \bmod 5\ ), x \equiv 2(\mod 7)$$