In class 5

1. Prove that

$$\forall n \geq 1, \sum_{j=1}^{n}(6j-1) = 3n^2 + 2n$$

$$5 + 11 + 17 + ... + 6n - 1 = 3n^2 + 2n$$

   by induction using the following steps:

   (a) Base step: show that it is true if $n = 1$

   (b) Write the inductive hypothesis, that is write what we assume if $n = k$

   (c) Write what we need if $n = k + 1$ and then prove it using the inductive hypothesis and very little algebra.

2. Prove that $\forall n \geq 1, n^3 - n$ is divisible by 6 by induction, using the following steps:

   (a) Base step: show true for $n = 1$

   (b) Write the inducive hypothesis, that is, write what we assume if $n = k$

   (c) Prove that the result is true for $n = k + 1$ assuming it is true for $n = k$
   hint: after doing the necessary algebra, you may want to subtract and add $3k$ to get the statement you want. Factoring will help to prove the second term is divisible by 2 as well as by 3.

   The Fibonacci sequence is defined by the recurrence

$$f_n = f_{n-1} + f_{n-2}, f_1 = f_2 = 1$$

3. Write the first 8 terms.

4. Prove by induction that

$$\sum_{j=1}^{n} f_{2j-1} = f_{2n}$$

$$f_1 + f_3 + f_5 + ... + f_{2n-1} = f_{2n}$$

   (a) Base step

   (b) Induction hypothesis

   (c) Prove true for $n = k + 1$

5. Prove by induction that

$$\sum_{j=1}^{n} f_j^2 = f_n f_{n+1}$$

$$f_1^2 + f_2^2 + f_3^2 + ... + f_n^2 = f_n f_{n+1}$$

(a) Base step

(b) Induction hypothesis

(c) Prove true for $n = k + 1$
hint: you can factor out a f_{k+1} from both terms in the left hand side.

6. Solve the equation $x^2 = x + 1$ using the quadratic formula.

Call the positive solution r

7. Prove by induction that $\forall n \geq 2, f_n \geq r^{n-2}$ by the following steps:

(a) Base step (note $n = 2$)

(b) Write the induction hypothesis.

(c) Prove that given the induction hypothesis it is true for $n = k + 1$
hint: you can reduce to the previous case by writing $f_{k+1} = f_k + f_{k-1}$, use the inductive hypotheses for both terms, factor out r^{k-3} and then use $r^2 = r+1$. Note that when you use the induction hypothesis for both k **and** $k - 1$ you are using Strong Induction, that is assuming the inductive hypothesis for all $n \leq k + 1$.