

Things to know about exponentials and logs:

- Properties of exponential functions: Domain and Range. Typical graph. Graph of  $f(x) = e^x$
- Properties of logarithmic functions: Examples such as  $f(x) = \log x$ ,  $f(x) = \ln x$  and their graphs.
- $y = \log_a x$  means  $a^y = x$
- The number  $e = \left(1 + \frac{1}{n}\right)^n$  as  $n \rightarrow \infty$
- Natural logarithm:  $\ln x = \log_e x$   $y = \ln x$  means  $e^y = x$
- Common logarithm  $\log x = \log_{10} x$   $y = \log x$  means  $10^y = x$
- Logarithms are the inverse of exponentials. That is,  
 $a^{\log_a x} = x$ ,  $\log_a a^x = x$

For example  $\ln e^x = x$ ,  $e^{\ln x} = x$  and  $\log 10^x = x$ ,  $10^{\log x} = x$

- Properties of logarithms.

$$\log uv = \log u + \log v$$

$$\log \frac{u}{v} = \log u - \log v$$

$$\log u^n = n \log u$$

For example:  $\log 2000 = \log 2 + \log 1000 = \log 2 + 3$

- Change of base formula:  $\log_a x = \frac{\log_b x}{\log_b a}$

For example:  $\log_9 27 = \frac{\ln 27}{\ln 9}$

Another example: Solve  $1.06^t = 2$   $t = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06}$

- Compound interest:  $A = P \left(1 + \frac{r}{n}\right)^{nt}$
- Continuous compounding:  $A = Pe^{rt}$
- Exponential growth and decay:  $A = A_0 e^{rt}$
- Important: Since  $e^{\ln x} = x$  and  $\ln x^n = n \ln x$  it follows that  $x^n = e^{n \ln x}$

In other words,  $A^B = e^{B \ln A}$

For example:  $2^{\sqrt{2}} = e^{\sqrt{2} \ln 2}$  and  $x^{\frac{1}{x}} = e^{\frac{\ln x}{x}}$