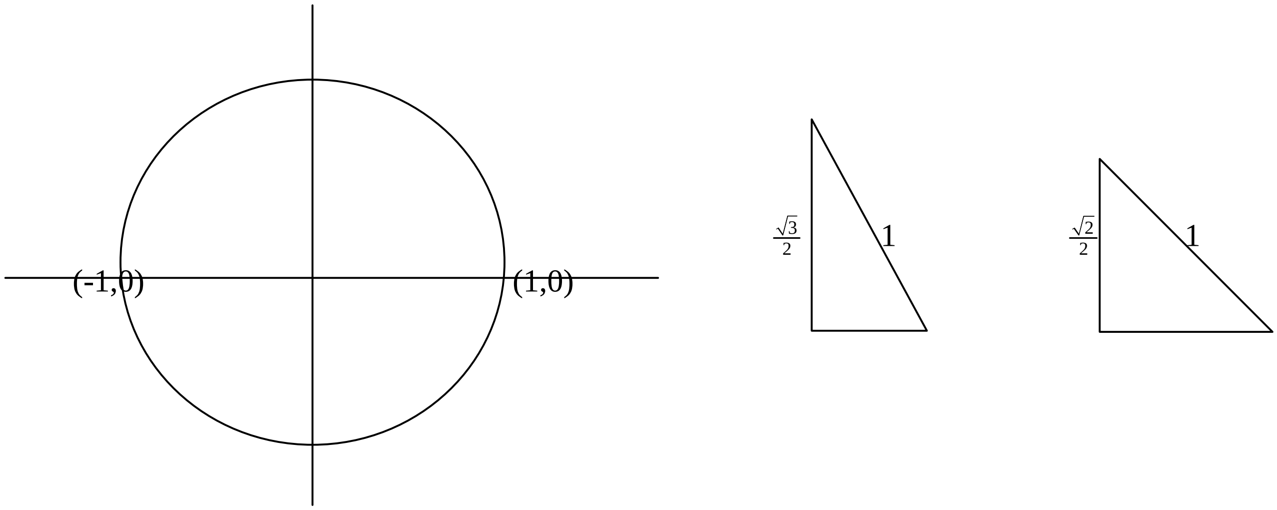



Have the above on your paper when you work. Label the angles of the triangles and the sides appropriately. Also label the points on the unit circle where the circle meets the axes.

Example. To find $\sin\left(\frac{2\pi}{3}\right)$ first locate $\frac{2\pi}{3}$ on the circumference of the circle. Remember the entire circumference is $2\pi$, half the circumference from (1,0) to (-1,0) is $\pi$, and therefore $\frac{2\pi}{3}$ is found by dividing the upper half into three equal parts and taking two of them. Using the reference triangle, locate and label the coordinates of the point. Make sure the hypotenuse of the triangle is the radius of the circle. In this example the coordinates are $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$. Sine is the second coordinate, cosine is the first. Tangent is the ratio of sine over cosine.

Because sine and cosine correspond to coordinates of points on the unit circle, it follows that $-1 \le \sin x \le 1$ and $-1 \le \cos x \le 1$. This says the range of sine and cosine is [-1,1]. Another way to write this is $|\sin x| \le 1$. It also follows that $\sin^2 x + \cos^2 x = 1$. This implies $\sin x = \pm\sqrt{1-\cos^2 x}$ etc.

Example: Suppose $\sin\theta = .6$, with $\theta$ in quadrant II. Then
$\cos\theta = -\sqrt{1-.6^2} = -\sqrt{1-.36} = -\sqrt{.64} = -.8$

Facts: Sine is odd, cosine is even, and tangent is odd.
The domain of sine and cosine is all real numbers.
The domain of tangent is all real numbers except $\frac{\pi}{2}+k\pi$ for integer *k*.

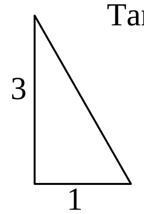
Example: If $\sin\theta = -.3$ then $\sin(-\theta) = .3$ Since $\cos\frac{\pi}{3} = \frac{1}{2}$ then $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

$\sin^{-1} x = \arcsin x$ is the number between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is *x*. This says the range of $\sin^{-1} x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Example: Find $\sin^{-1} 0$. Since $\sin 0 = 0, \sin^{-1} 0 = 0$

Example: Find $\sin^{-1} -\frac{\sqrt{3}}{2}$. You are looking for the angle whose sine is $-\frac{\sqrt{3}}{2}$. Find the triangle that has $\frac{\sqrt{3}}{2}$ as one side, orient it in the circle so that that side is the vertical distance below the horizontal axis. Stick to the right half of the circle because $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$. The angle you are looking for is $-\frac{\pi}{3}$

Example: Find sin(arctan3). You are looking for the sine of the angle whose tangent is 3. A triangle will help. Tangent is opposite over adjacent, so label as follows:

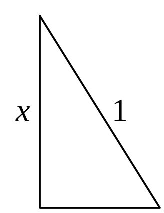



Now find the hypotenuse using Pythagoras and you are done. Hypotenuse is $\sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$ so sin(arctan3)= $\frac{3}{\sqrt{10}}$

Example: Find sin(arctanx). Just like above, only with the 3 replaced by an x. Thus the hypotenuse is $\sqrt{x^2 + 1}$ and so sin(arctanx) is $\frac{x}{\sqrt{x^2 + 1}}$

Example: Find cos(arcsinx). Same triangle, different label. We want the cosine of the angle whose sine is x. At this point it should be obvious, but lets draw a triangle. sine is opposite over hypotenuse, so we label as follows.

The adjacent side is now $\sqrt{1 - x^2}$ and therefore cos(arcsinx)=$\sqrt{1 - x^2}$




Note: We knew this already. Since $\sin^2 \theta + \cos^2 \theta = 1$ it follows that $\cos \theta = \pm\sqrt{1 - \sin^2 \theta}$ and $\arcsin x = \theta \Leftrightarrow \sin \theta = x$

Example: Find the other 4 trig function of $\phi$ if $\sec \phi = \frac{13}{5}, \sin \phi = -\frac{12}{13}$.

Solution. No particular need for a triangle, although it wouldn't hurt. We know $\sec \phi = \frac{1}{\cos \phi}$ so if $\sec \phi = \frac{13}{5}$ then $\cos \phi = \frac{5}{13}$. Now $\tan \phi = \frac{\sin \phi}{\cos \phi} = -\frac{12}{5}$. Convince yourself that you need not worry about the denominators (they cancel) but you do need to worry about the minus sign. Cotangent it the reciprocal of tangent and cosecant is the reciprocal of secant, so just flip it.

You should know the graphs of sine, cosine and tangent. Also the period, amplitude and phase shift for $y = a\sin(bx + c)$