Worksheet on sigma notation

1. Write out the sum $\sum_{k=1}^{5} 2k$
2. Using the distributive law, show that it is the same as $2\sum_{k=1}^{5} k$
3. What is $\sum_{k=1}^{5} 1$? How about $\sum_{k=1}^{n} 1$? More generally $\sum_{k=1}^{n} c$ where c is any constant?
4. Write out the sum $\sum_{1}^{5} k^2 + k$
5. Using the commutative law, show that this is the same as $\sum_{k=1}^{5} k^2 + \sum_{k=1}^{5} k$
6. Express in sigma notation: $1-3+5-7+9-11+13-15+17$
7. Observe that $1 = 1^2, 1+3 = 2^2, 1+3+5 = 3^2, 1+3+5+7 = 4^2$
8. What is the next sum? Express this relationship in sigma notation.
9. Show that $\sum_{k=1}^{n} (k^2 - (k-1)^2) = n^2$ (Don't be confused by this; the "n" is the upper limit. Write out the first few terms without computing and you will see that the sum "telescopes".)
10. Note that $k^2 - (k-1)^2 = 2k - 1$ (This has nothing to do with sums, this is elementary algebra.)
11. Conclude that $\sum_{k=1}^{n} (2k-1) = n^2$ (Not much work here, just put in the equal sign.)
12. Now show that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ by the following method: Rewrite $\sum_{k=1}^{n} (2k-1) = n^2$ as $2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = n^2$ and solve for $\sum_{k=1}^{n} k$
13. Express in sigma notation: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ and compute this number.
14. Compute the number $\sum_{k=1}^{5} \frac{1}{2^k}$

15. Compute the number $\sum_{k=1}^{6} \frac{1}{2^k}$

16. What do you guess $\sum_{k=1}^{7} \frac{1}{2^k}$ will be?

17. Assuming we had a good definition for an infinite sum, what should $\sum_{k=1}^{\infty} \frac{1}{2^k}$ be?

18. Using the formulas

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Compute $\sum_{k=1}^{20} k(k-2)$ and $\sum_{k=1}^{10} k(k+1)(k+2)$

19. Try computing this without getting confused between the index *k* and the upper limit *n* (which has nothing to do with the index, it is a constant).

$$\sum_{k=1}^{n} \frac{k}{n^2}$$

This is a harder set of problems, analogous to numbers 9 – 12.

20. Show that $\sum_{k=1}^{n} k^3 - (k-1)^3 = n^3$

21. Using elementary algebra, show $k^3 - (k-1)^3 = 3k^2 - 3k + 1$

22. Conclude that $\sum_{k=1}^{n} 3k^2 - 3k + 1 = n^3$

23. Solve the above equation for $\sum_{k=1}^{n} k^2$