Week 2 Notes
Here are some ordered pairs for the graph of $y = 2^x$

$$f(x) = 2^x : \{(-3, \frac{1}{8}), (-2, \frac{1}{4}), (-1, \frac{1}{2}), (0, 1), (1, 2), (2, 4), (3, 8)\}$$

This is a one to one function, which we can find interchanging the ordered pairs so x becomes y and y becomes x

$$f^{-1}(x) : \{(\frac{1}{8}, -3), (\frac{1}{4}, -2), (\frac{1}{2}, -1),), (1, 0), (2, 1), (4, 2), (8, 3)\}$$

So for example $f^{-1}(4) = 2$ and $f^{-1}(\frac{1}{4}) = -2$
Notice the only way to evaluate $f^{-1}(x)$ is to look at the table, or to answer this question $f^{-1}(x) = y$ solve $2^y = x$ There is no algebra to find the values of f^{-1}
If you recall how to find an inverse function, usually you switch x and y and solve for y. But if we try it with $y = 2^x$ and switch to get $x = 2^y$ we are stuck right away because there is no algebra to solve this for y. On the other hand it is clear that $f(x) = 2^x$ is a one to one function, and it does have an inverse $f^{-1}(x)$ which is denoted $f^{-1}(x) = \log_2(x)$ read "log base two of x" . The language makes sense because 2 is the base. But we can only evaluate $\log_2(x)$ for specific x by solving $2^y = x$

So for example $\log_2(16) = 4$ because $2^4 = 16$ and $\log_2\left(\frac{1}{\sqrt{8}}\right) = -\frac{3}{2}$ since $2^{-\frac{3}{2}} = \frac{1}{\sqrt{8}}$

At the moment if we want to find $\log_2(100)$ we are out of luck because we do not know how to solve $2^y = 100$ other than saying $y = \log_2(100)$

In general, the inverse of an exponential function $f(x) = b^x$ is called $\log_b(x)$ and

$$\log_b(x) = y \iff y = b^x$$

1. $3^2 = 9 \iff \log_3(9) = 2$
2. $5^{-2} = \frac{1}{25} \iff \log_5(\frac{1}{25}) = -2$
3. $8^{\frac{1}{3}} = 2 \iff \log_8(2) = \frac{1}{3}$
4. $b^0 = 1 \iff \log_b(1) = 0$

The last example shows that the log of 1 is 0 irrespective of the base.

There are two exceptions to this notation. log base 10 is written without a base, i.e. $\log_{10}(x)$ is just written $\log(x)$

1. $\log(100) = 2 \iff 10^2 = 100$
2. $\log(-.01) = -2 \iff 10^{-2} = .01$
3. $\log(10000000000) = 10 \iff 10^{10} = 10000000000$
4. $\log(10^{25}) = 25 \iff 10^{25} = 10^{25}$

The last example shows that $\log_b(b^x) = x$ because $b^x = b^x$ although the simple reason is that $\log_b(x)$ is the inverse of b^x and it is always true that $f^{-1}(f(x)) = x$
The other exception is the natural log, the inverse of the natural exponential function.

$$\log_e(x) = \ln(x)$$

1. $\ln(1) = 0 \iff e^0 = 1$
2. $\ln(e^3) = 0 \iff e^3 = e^2$
3. $\ln(\sqrt{e}) = \frac{1}{2} \iff e^{\frac{1}{2}} = \sqrt{e}$

In general we don't know how to find $\ln(x)$ except using a calculator, it is on every calculator (almost) and certainly we can use wolfram.