Week 1 Notes

The distributive property and exponential functions

The distributive property is usually written

$$a(b+c) = ab + ac$$

and in (math) English we say "multiplication distributes over addition". This is the link between multiplication and addition. Because the equal sign reads from left to right and also from right to left (we say equality is "symmetric") , and because both multiplication and addition are commutative, there are several different ways to write this.

$$a(b+c) = ab + ac \tag{1}$$

$$(b+c)a = ba + ca \tag{2}$$

$$ab + ac = (b+c)a \tag{3}$$

$$ca + ab = a(c+b) \tag{4}$$

and so on

1. A typical example from arithmetic using (4) might be

   $3 \times 179 + 179 \times 7 = (3+7)179 = 1790$

2. From algebra using (1) $3(x+7) = 3x + 21$ (math teachers say"multiply it out")

3. From algebra using (3) $7x - 2x = (7-2)x = 5x$ "combine like terms"

4. (4) $x^2 + 3x = x(x+3)$ "factoring"

All are the distributive law in action.

For our purposes in 162 and exponential functions, consider the following (easy) question. Using (3) with $a = 90, b = 1, c = .08$ we get

1. $$90 \times 1 + 90 \times .08 = (1 + 0.08) \times 90 = (1.08) \times 90 = 97.2$$

   As a word problem, if you purchase an book for \$ 90 and have to pay 8% tax, then the total price of the book will be \$ 97.2. In other words, to increase a number by 8% multiply it by 1.08

2. To increase a number P by 15% multiply it by 1.15 . Another way to think about it is 100% + 15% = 115% =1.15

3. To increase a number by 50% multiply it by 1.5

4. to **decrease** a number by 10% multiply it by 0.9.

   By the distributive property $P - 0.1P = (1 - .1)P = .9P$ or think "100% - 10% = 90% = 0.9"

5. To decrease a number by 15% multiply it by .85

   And so on

6. Increase \$ 1,000 by 20% and then increase the result by 10%. First we increase by 20% to get \$1,200, then increase that by 10% to get \$1,320. Notice that increasing by 20% and then 10% is NOT a 30% increase. In fact it is a 32% increase because $(1.2)(1.1) = 1.32$

7. After a 20% discount, a pair of shoes cost \$112. What was the price before the discount?

   (a) Wrong method: 20% of 112 is 22.4, then 112 + 22.4 = 134.4 This has to be incorrect, since 20% of the sale price is necessarily less than 20% of the original price, and so adding it back will not give the original amount

   (b) Correct method: To decrease a number by 20% is the same as multiplying by 0.8. We don't know the original price, but if we call it P we have $0.8P = 112 \iff P = 112 \div 0.8 = 140$

8. Invest \$1,000 at 4% compounded annually. How much will you have in 15 years?

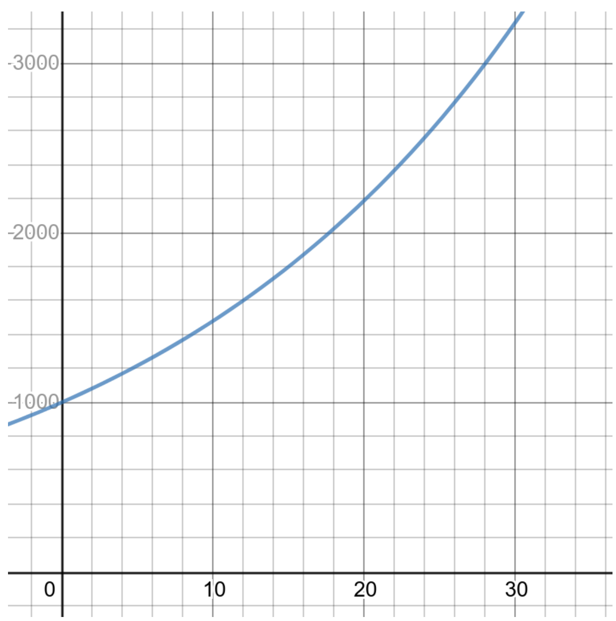
(a) In one year you will have $1,000 \times 1.04 = 1040$

(b) In two years you will have $1040 \times 1.04 = 1000 \times 1.04 \times 1.04 = 1000 \times 1.04^2 = 1081.6$

(c) In three years you will have $1000 \times (1.04)^3$

(d) In 15 years you will have $1000 \times (1.04)^{15}$ which is why you need a calculator **Wolf**

(e) In t years you will have $f(t) = 1000 \times (1.04)^t$ our first exponential function.



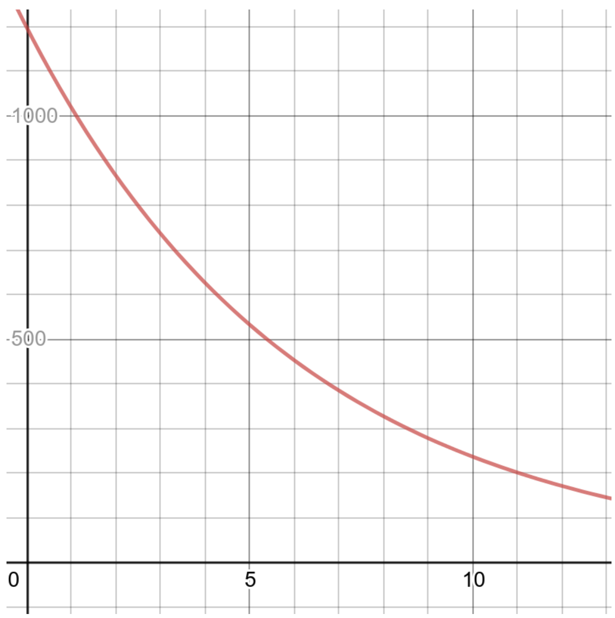
As a mathematical function the domain would be $\mathbb{R}$ but since we are talking about time I started the graph close to 0. The picture suggests that your investment would double to \$2,000 in approximately 18 years.

9. You purchase a used car for \$1200. A year later it is worth \$1020. What will it be worth in 5 years?

   (a) The linear model: it loses $1200-1020 = 180$ a year, so in year one (we start at year 0) it is worth \$1020 and in year two $1020-180 = 840$ in year 5 $1200-5\times 180 = 300$

   (b) The more sophisticated exponential model: It lost 15% of its value, since $\frac{180}{1200} = .15 = 15\%$ so it will lose 15% of its value each year. To reduce a number by 15% is the same as multiplying by .85. In year one: $1200 \times .85 = 1020$. In year 2, $1200 \times .85^2 = 869$ and in year 5 $1200 \times .85^5 = 532.45$ Notice we did not have to compute the percent decrease, we would know to multiply by .85 because $\frac{1020}{1200} = .85$

   (c) In t years it would be worth $f(t) = 1200 \times .85^t$ a decreasing function.



Notice that this decreasing function never goes below 0, so your car always has some value. Also notice since it is strictly decreasing, making it a one to one function.