152 Test 3 Name: ________________________

1. Use the formula

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + ... = \frac{a}{1-r}$$

to add the following:

$$\frac{1}{3} + \frac{1}{3} \times \left(\frac{2}{3}\right)^2 + \frac{1}{3} \times \left(\frac{2}{3}\right)^4 + \frac{1}{3} \times \left(\frac{2}{3}\right)^6 + \frac{1}{3} \times \left(\frac{2}{3}\right)^8 + ... = \sum_{k=0}^{\infty} \frac{1}{3} \left(\frac{2}{3}\right)$$

2. Homer and Jethro take turns rolling on die. Whoever gets a one or a two first wins. If Homer throws first, what it the probability that he wins?

3. Solve the quadratic equation $x = \frac{3}{7} + \frac{4}{7}x^2$

4. Wilma and Betty play a series of games. The probability Wilma wins each game is $\frac{3}{7}$. What is the probability that Wilma will ever be up by one game?

5. What is the probability Wilma will ever be up by 5 games?

6. Fred and Barny toss a coin. If it shows Heads, Fred pays Barny one dollar but if it shows tails, Barny pays Fred one dollar. If Fred has \$10 and Barney has only \$5, what is the probability that Fred wins all the money?

7. Suppose Barney has fixed the coin so that it shows Heads with probability $\frac{4}{5}$. What is the probability Fred wins all the money?

8. Suppose on average you receive 2 pieces of junk mail every day.

   Assuming that junk mail is delivered in a Poisson distribution, what is the probability you get no junk mail tomorrow?

9. What is the probability you get two or more pieces of junk mail tomorrow?

10. For the folling data: $\{2, 2, 2, 3, 4, 5, 5, 5, 6, 9\}$

    (a) Find the mean.

    (b) Find the variance.

    (c) Find the standard deviation.

Some useful formulas: Unlimited credit: Peter extends Paul unlimited credit, and the probability Peter wins is p then the probability Peter is ever up by k games is

$$h = \begin{cases} 1 & \text{if } \; p \geq \frac{1}{2} \\ \left(\frac{p}{1-p}\right)^k & \text{if } \; p < \frac{1}{2} \end{cases}$$

Limited credit: If Peter starts with s and the total is t then put $r = \frac{1-p}{p}$ and the probability Peter wins all the money before going broke is

$$h = \begin{cases} \dfrac{s}{t} & \text{if } \; p = \frac{1}{2} \\ \dfrac{1-r^s}{1-r^t} & \text{if } \; p \neq \frac{1}{2} \end{cases}$$

Poisson process with expected value λ $P(x = k) = \frac{\lambda^k e^{-\lambda}}{k!}$,
specifically $P(x = 0) = e^{-\lambda}$, $P(x = 1) = \lambda e^{-\lambda}$, $P(x = 2) = \frac{\lambda^2 e^{-\lambda}}{2}$